

# One-loop corrections to the Drell-Yan process in SANC (I). The charged current case.

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## Abstract

Radiative corrections to the charged current Drell-Yan processes are revisited. Complete one-loop electroweak corrections are calculated within the automatic SANC system. Electroweak scheme dependence and the choice of the factorization scale are discussed. Comparisons with earlier calculations are presented.

*Key words:* Drell-Yan process, electroweak radiative corrections, helicity amplitudes

*PACS:* 13.85.Qk Inclusive production with identified leptons, photons, or other nonhadronic particles, 12.15.Lk Electroweak radiative corrections

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## 1 Introduction

Precision studies of the Drell-Yan process are vitally important for high energy hadronic colliders. This process provides information about weak interactions and contributes to the background to many of the searches for physics beyond the Standard Model. One-loop QED and electroweak (EW) radiative corrections (RC) to the Drell-Yan process at high energy hadronic collider were calculated by several groups in the past, see papers [1,2,3,4,5,6] and references therein. In this paper we present the results for the corrections to the charged current Drell-Yan process, obtained within the automatized system SANC [7,8] and some comparisons with earlier calculations. Starting from the construction of helicity amplitudes and EW form factors, SANC performs calculation of the process cross section and produces computer codes, which can be further used in the experimental data analysis.

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## 2 Preliminaries and Notation

Let us start with the partonic level, where we will consider interactions of *free* quarks (partons). The differential Born-level cross section of the process

$$\bar{d}(p_1) + u(p_2) \rightarrow l^+(p_4) + \nu_l(p_3) \quad (1)$$

in the center-of-mass system of the initial quarks reads

$$\frac{d\hat{\sigma}_0}{d\hat{\Omega}} = \frac{1}{4} \frac{1}{N_c} |V_{ud}|^2 \frac{G_F^2 M_W^2}{2\pi \hat{s}} \frac{\hat{u}^2}{(\hat{s} - M_W^2)^2 + \Gamma_W^2(\hat{s}) M_W^2}, \quad (2)$$

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{u} = (p_1 - p_3)^2,$$

where  $N_C = 3$  is the number of colors;  $V_{ud}$  is the relevant element of the CKM matrix;  $G_F$  is the Fermi coupling constant;  $M_W$  and  $\Gamma_W$  are the mass and the width of the  $W$ -boson, respectively.

## 3 Radiative Corrections at the Partonic Level

In order to get a more accurate description of the process we should go beyond the Born approximation and take into account different sources of radiative corrections. Here we will consider only EW contributions to the corrections, while effects of higher order QCD contributions (and *mixed* effects) are left beyond the scope of our study.

As usually, we subdivide the EW RC into the virtual (loop) ones, the ones due to soft photon emission, and the ones due to hard photon emission. An auxiliary parameter  $\bar{\omega}$  separates the soft and hard photonic contributions.

In the automatized system [8], the virtual corrections are accessible via menu chain **SANC** → **EW** → **Processes** → **4 legs** → **4f** → **Charged Current** → **f1 f1'** → **f f' (FF)**. The module, loaded at the end of this chain computes online the scalar form factors of the partonic sub-process (1). The parallel module **...f1 f1' → f f' (HA)** provides the relevant helicity amplitudes. For more details see section 2.5 of SANC description [7] and the book [9].

The real photon emission process

$$\bar{d}(p_1) + u(p_2) \rightarrow l^+(p_4) + \nu_l(p_3) + \gamma(p_5) \quad (3)$$

should be taken into account as well. In order to get a fast computer code, we had to perform integration of the hard photon emission contribution analytically. But we had to keep the

possibility to impose experimental cuts. Now we have the two branches. The first one contains the complete chain of analytical integrals over the hard photon phase space. It provides the double-differential distribution  $d^2\hat{\sigma}_{\text{hard}}/(dc d\hat{s}')$  and the single differential distribution  $d\hat{\sigma}_{\text{hard}}/dc$ , where  $c = \cos \angle(\vec{p}_2\vec{p}_4)$  and  $\hat{s}' = (p_3 + p_4)^2$ . The second branch provides the double-differential distribution  $d^2\hat{\sigma}_{\text{hard}}/(dc dM_x^2)$ , where  $M_x^2 = 2p_3p_5$  which is directly related to the charged lepton energy in the center-of-mass system of the initial quarks:

$$\hat{E}_\mu = p_4^0 = \frac{\hat{s} + m_l^2 - M_x^2}{2\sqrt{\hat{s}}}. \quad (4)$$

The differential distributions of the tree-level radiative process  $\bar{d} + u \rightarrow l^+ + \nu_l + \gamma$  were compared with the corresponding distributions obtained by means of the CompHEP package [10]. Cross section distributions in the cosine of the outgoing charged lepton (the muon is used) angle and in the lepton energy are considered. 20 bins are constructed for each of the distributions. Bins in the muon energy are

$$(n_{\text{bin}} - 1) \times 5 \text{ GeV} < E_\mu < n_{\text{bin}} \times 5 \text{ GeV}. \quad (5)$$

The cut on the muon energy ( $E_\mu < 95 \text{ GeV}$ ) is imposed in both distributions to avoid the region with soft photons, where CompHEP is not supposed to work well. Angular bins are

$$-1 + \frac{n_{\text{bin}} - 1}{10} < c < -1 + \frac{n_{\text{bin}}}{10}. \quad (6)$$

An agreement was found as can be seen from Table 1

Using the splitting of the  $W$ -boson propagators in the case of real photon emission off the virtual  $W$ , we separate the contributions of the initial state radiation, the final state one, and their interference in a gauge invariant way [11]. The splitting is introduced by the following formula:

$$\frac{1}{\hat{s} - (M_W + i\Gamma_W)^2} \cdot \frac{1}{\hat{s}' - (M_W + i\Gamma_W)^2} = \frac{1}{(\hat{s} - \hat{s}')} \left( \frac{1}{\hat{s}' - (M_W + i\Gamma_W)^2} - \frac{1}{\hat{s} - (M_W + i\Gamma_W)^2} \right). \quad (7)$$

In the center-of-mass system  $(\hat{s} - \hat{s}') = 2p_5^0\sqrt{\hat{s}}$ . The fixed  $W$ -width scheme is used here and in what follows.

For the two choices of variables we have simple analytical expressions for the corresponding soft photon contributions. The infrared singularities in them are regularized by the auxiliary photon mass. The energy of a soft photon is limited from above by a cut in the integral either over  $\hat{s}'$  or over  $M_x^2$ .

bin	bins in $E_\mu$		bins in $c$	
	SANC	CompHEP	SANC	CompHEP
1	0.0006(1)	0.0006(1)	0.5867(1)	0.5869(2)
2	0.0010(1)	0.0010(1)	0.3538(1)	0.3537(1)
3	0.0023(1)	0.0023(1)	0.1857(1)	0.1858(1)
4	0.0452(1)	0.0451(1)	0.1111(1)	0.1111(1)
5	0.0569(1)	0.0569(1)	0.0740(1)	0.0741(1)
6	0.0549(1)	0.0549(1)	0.0541(1)	0.0541(1)
7	0.0546(1)	0.0546(1)	0.0427(1)	0.0428(1)
8	0.0563(1)	0.0563(1)	0.0360(1)	0.0361(1)
9	0.0603(1)	0.0603(1)	0.0321(1)	0.0321(1)
10	0.0667(1)	0.0666(1)	0.0298(1)	0.0297(1)
11	0.0755(1)	0.0755(1)	0.0287(1)	0.0287(1)
12	0.0868(1)	0.0867(1)	0.0284(1)	0.0285(1)
13	0.1008(1)	0.1008(1)	0.0286(1)	0.0287(1)
14	0.1175(1)	0.1175(1)	0.0292(1)	0.0292(1)
15	0.1372(1)	0.1372(1)	0.0299(1)	0.0299(1)
16	0.1605(1)	0.1605(1)	0.0302(1)	0.0301(1)
17	0.1881(1)	0.1881(1)	0.0294(1)	0.0293(1)
18	0.2227(1)	0.2227(1)	0.0263(1)	0.0263(1)
19	0.2739(1)	0.2740(2)	0.0187(1)	0.0187(1)
20	0.0	0.0	0.0059(1)	0.0059(1)

Table 1

Bin by bin comparison of differential distributions for the process  $\bar{d} + u \rightarrow \mu^+ + \nu_\mu + \gamma$ .

In the course of calculations of the  $\mathcal{O}(\alpha)$  corrections we met the so-called *on-shell* singularities, which appear in the form of  $\ln(\hat{s} - M_W^2 + i\epsilon)$ . As was shown in detail in Ref. [3], they can be regularized by the  $W$ -width:

$$\ln(\hat{s}' - M_W^2 + i\epsilon) \rightarrow \ln(\hat{s}' - M_W^2 + iM_W\Gamma_W). \quad (8)$$

In the analytical formulae for radiative corrections one can find logarithms with quark and lepton mass singularities:

$$\ln \frac{\hat{s}}{m_l^2}, \quad \ln \frac{\hat{s}}{m_u^2}, \quad \ln \frac{\hat{s}}{m_d^2}. \quad (9)$$

In the experimental set-up with calorimetric registration of the final state charged particles (typical for electrons), the lepton mass singularity cancels out in the result for the correction to an observable cross section in accordance with the Kinoshita–Lee–Nauenberg theorem [12,13]. But if the experiment is measuring the energy of the charged lepton without summing it with the energies of accompanying collinear photons (typical for muons), the logarithms with the lepton mass singularity remain in the result and give a considerable numerical contribution. Re-summation of these logs in higher orders was discussed in Refs. [14,15,16].

### 3.1 Treatment of Quark Mass Singularities

One-loop radiative corrections contain terms proportional to the logarithms of the quark masses,  $\ln(\hat{s}/m_{u,d}^2)$ . They come from the initial state radiation contributions including hard, soft and virtual photon emission. Such initial state mass singularities are well known, for instance, in the process of  $e^+e^-$  annihilation. But in the case of hadron collisions these logs have been already *effectively* taken into account in the parton density functions (PDF's). In fact in the procedure of PDF's extraction from the experimental data, QED radiative corrections (to the quark line) have not been systematically subtracted. Therefore the present PDF's effectively include not only the QCD evolution but also the QED one. Moreover, it is worth to note that the leading log behavior of the QED and QCD DGLAP evolution of quark density functions are proportional to each other. So one gets evolution of PDF's with an effective coupling constant

$$\alpha_s^{\text{eff}} \approx \alpha_s + \frac{Q_i^2}{C_F} \alpha, \quad (10)$$

where  $\alpha_s$  is the strong coupling constant,  $\alpha$  is the fine structure constant,  $Q_i$  is the quark charge, and  $C_F$  is the QCD color factor. The nontrivial difference between the QED evolution and the QCD one starts to appear from higher orders, and the corresponding numerical effect is small compared to the remaining QCD uncertainties in PDF's [17,18,19,20]. The best approach to the whole problem would be to re-analyze all the experimental DIS data taking into account QED corrections to the quark line at least at the next-to-leading order. But for the present moment we can limit ourselves with application of a certain subtraction scheme to the QED part of the radiative corrections for the process under consideration. We will use here the  $\overline{\text{MS}}$  scheme [21], the DIS scheme can be used as well. This leads to avoid the double counting of the initial quark mass singularities in our result for the corrections to the partonic cross section and in the corresponding PDF. The latter should be also taken in the same scheme with the same factorization scale.

In fact, using the initial condition for the non-singlet NLO QED quark structure function, which coincides with the QCD one with the trivial substitution  $C_F \alpha_s \rightarrow Q_i^2 \alpha$ , see Ref. [22],

one gets the following expression for the terms to be subtracted from the full calculation with massive quarks:

$$\delta^{\overline{\text{MS}}} = \sum_{i=1,2} Q_i^2 \frac{\alpha}{2\pi} \int_0^1 d\xi_i \left[ \frac{1 + \xi_i^2}{1 - \xi_i} \left( \ln \frac{M^2}{m_i^2} - 2 \ln(1 - \xi_i) - 1 \right) \right]_+ \hat{\sigma}_0(\xi_i), \quad (11)$$

where  $Q_i$  and  $m_i$  denote the charge and the mass of the given quark;  $M$  is the factorization scale;  $\hat{\sigma}_0(\xi_i)$  is the cross section at the partonic level with the reduced value of the quark momentum:  $p_i \rightarrow \xi_i p_i$ . The subtracted cross section with  $\mathcal{O}(\alpha)$  corrections is given by

$$\hat{\sigma}_1^{\overline{\text{MS}}} = \hat{\sigma}_1 - \delta^{\overline{\text{MS}}}. \quad (12)$$

Then it can be convoluted with PDF's as shown below in Eq. (15).

But there is an alternative way to perform the subtraction. Really, to avoid the double counting of the quark mass singularities, we can leave them in the corrected cross section, but remove from the PDF's:

$$\begin{aligned} \bar{q}(x, M^2) &= q(x, M^2) - \int_x^1 \frac{dz}{z} q\left(\frac{x}{z}, M^2\right) \frac{\alpha}{2\pi} Q_q^2 \left[ \frac{1+z^2}{1-z} \left\{ \ln\left(\frac{M^2}{m_q^2}\right) - 2 \ln(1-z) - 1 \right\} \right]_+ \\ &\equiv q(x, M^2) - \Delta q, \end{aligned} \quad (13)$$

where  $q(x, M^2)$  can be taken directly from the existing PDF's in the  $\overline{\text{MS}}$  scheme (see Ref. [3] for the corresponding formula in the DIS scheme). It can be shown analytically (see i.e. Ref. [3]), that this procedure is equivalent to the subtraction from the cross section, and that it really removes (hides) the dependence on the quark masses. The advantage of the last approach is that it can be used regardless of the way to represent the partonic cross section: it can be kept even in the completely differential form.

The *natural* choices of the factorization scale are  $M^2 = M_W^2$  (when the returning to the  $W$ -resonance is allowed by kinematical cuts) and  $M^2 = \hat{s} = x_1 x_2 s$ . Variations with respect to the choice should be studied.

In order to avoid the appearance of spurious higher order terms for the case of subtraction from PDF's, we suggest to apply a procedure of *linearization*. Schematically it can be represented as follows

$$\begin{aligned} \bar{q}_1(x_1, M^2) \times \bar{q}_2(x_2, M^2) \times \hat{\sigma}_1 &= [q_1(x_1, M^2) - \Delta q_1] \times [q_2(x_2, M^2) - \Delta q_2] \times (\hat{\sigma}_{\text{Born}} + \hat{\sigma}_\alpha) \\ &\rightarrow q_1(x_1, M^2) \times q_2(x_2, M^2) \times \hat{\sigma}_{\text{Born}} + q_1(x_1, M^2) \times q_2(x_2, M^2) \times \hat{\sigma}_\alpha \\ &\quad - [q_1(x_1, M^2) \times \Delta q_2 + q_2(x_2, M^2) \times \Delta q_1] \times \hat{\sigma}_{\text{Born}}, \end{aligned} \quad (14)$$

where  $\hat{\sigma}_{\text{Born}}$  and  $\hat{\sigma}_\alpha$  denote the Born-level partonic cross section and the  $\mathcal{O}(\alpha)$  RC contribution to it, respectively. Without the linearization procedure, terms with quark mass singularities would remain in the  $\mathcal{O}(\alpha^2)$  contribution to the cross section.

## 4 Radiative Corrections to Hadronic processes

The double-differential cross section of the Drell-Yan process can be obtained from the convolution of the partonic cross section with the quark density functions:

$$\frac{d\sigma_{\text{RC}}^{pp \rightarrow \mu^+ \nu X}(s)}{dc dE_\mu} = \sum_{q_1 q_2} \int_0^1 dx_1 \bar{q}_1(x_1, M^2) \int_0^1 dx_2 \bar{q}_2(x_2, M^2) \frac{d^2 \hat{\sigma}^{q_1 q_2 \rightarrow \mu^+ \nu}(\hat{s})}{d\hat{c} d\hat{E}_\mu} \mathcal{J} \Theta(c, E_\mu), \quad (15)$$

where the parton densities with *bars* mean the ones modified by the subtraction of the quark mass singularities; the step function  $\Theta(c, E_\mu)$  defines the phase space domain corresponding to the given event selection procedure. The partonic cross section is taken in the center-of-mass reference frame of the initial quarks, where the cosine of the muon scattering angle,  $\hat{c}$ , and the muon energy,  $\hat{E}_\mu$ , are defined. The transformation into the observable variables  $c$  and  $E_\mu$  involves the Jacobian:

$$\begin{aligned} \mathcal{J} &= \frac{\partial \hat{c}}{\partial c} \cdot \frac{\partial \hat{E}_\mu}{\partial E_\mu} = \frac{4x_1 x_2}{a^2} \cdot \sqrt{\frac{a^2(1+c)}{x_1[a+x_2(1+c)]}}, \\ a &= x_1 + x_2 - c(x_1 - x_2), \quad \hat{c} = 1 - (1-c)\frac{2x_1}{a}, \\ \hat{s} &= sx_1 x_2, \quad \hat{E}_\mu = \frac{\sqrt{\hat{s}}}{2}, \quad \hat{E}_\mu = E_\mu \sqrt{\frac{1-c^2}{1-\hat{c}^2}}. \end{aligned} \quad (16)$$

## 5 Numerical Results and Conclusions

For numerical evaluations we take the same set of input parameters as in Ref. [5]:

$$\begin{aligned}
\alpha &= 1/137.0359895, & \alpha(M_Z^2) &= 1/128.887, & G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \\
M_W &= 80.35 \text{ GeV}, & M_Z &= 91.1867 \text{ GeV}, & M_H &= 150 \text{ GeV}, \\
\Gamma_W &= 2.08699 \text{ GeV}, & \alpha_s &= 0.119, \\
m_e &= 0.51099907 \text{ MeV}, & m_\mu &= 105.658389 \text{ MeV}, & m_\tau &= 1.77705 \text{ GeV}, \\
m_u &= 4.85 \text{ MeV}, & m_c &= 1.55 \text{ GeV}, & m_t &= 174.17 \text{ GeV}, \\
m_d &= 4.85 \text{ MeV}, & m_s &= 150 \text{ MeV}, & m_b &= 4.5 \text{ GeV}, \\
|V_{ud}| &= 0.975, & |V_{us}| &= 0.222, \\
|V_{cd}| &= 0.222, & |V_{cs}| &= 0.975,
\end{aligned} \tag{17}$$

In Table 2 we present the results for the total cross section<sup>2</sup> of the process  $u + \bar{d} \rightarrow \nu_l + l^+ (+\gamma)$ . For the Born-level cross section we completely (in all listed digits) agree with the numbers given in Ref. [5]. The results for the radiative corrections with  $\overline{\text{MS}}$  subtraction (with factorizations scale being equal to  $M_W$ ) are also in a fair agreement. Small differences there can be due to different treatment of EW scheme with respect induced higher order effects. Huge positive corrections in the case without subtraction of quark mass singularities above the  $W$ -peak are due to the initial state radiation which provides the radiative return to the resonance.

$\sqrt{s}/\text{GeV}$	40	80	120	200	500	1000	2000
$\hat{\sigma}_0/\text{pb}$	2.646	7991.4	8.906	1.388	0.165	0.0396	0.00979
$\delta/\%$ , full	-1.76	-7.88	92.9	129.6	161.2	172.5	178.9
$\delta/\%$ , $\overline{\text{MS}}(s)$	0.56	2.48	-17.2	-16.0	-17.9	-23.2	-31.5
$\delta/\%$ , $\overline{\text{MS}}(M_W)$	0.73	2.48	-12.9	-3.2	11.6	18.8	22.8
$\delta/\%$ , [5]	0.7	2.42	-12.9	-3.3	12	19	23

Table 2

The total lowest-order parton cross section  $\hat{\sigma}_0$  in the  $G_F$  EW scheme and the corresponding relative one-loop correction  $\delta$ .

The effect of EW scheme dependence is illustrated by Table 3. Results for the total partonic cross section at the Born and  $\mathcal{O}(\alpha)$  levels are given for two EW schemes. At the Born level the 7.3% difference appears just due to the difference in the definition of EW constants in the  $G_F$  and in the  $\alpha(0)$  schemes. As it should be the difference between the corrected cross sections is less than the one at the Born level. But still it is large and comparable with the ordered precision of the calculation. Certainly, usage of the  $\alpha(0)$  is not well motivated for the given energy range. And the difference  $\delta_1$  gives only an upper estimate of the uncertainty due to the EW scheme dependence. In any case we are going to perform further studies of this effect.

Table 4 represents the dependence of the hadronic Drell–Yan cross section on the values of

<sup>2</sup> factor  $|V_{ud}|^2$  has been dropped in the sake of comparison with Ref. [5].



$\sqrt{\hat{s}}/\text{GeV}$	40	80	120	200	500	1000	2000
$\hat{\sigma}_0/\text{pb}, [G_F]$	2.646	7991.4	8.906	1.388	0.165	0.0396	0.00979
$\hat{\sigma}_0/\text{pb}, [\alpha(0)]$	2.454	7410.2	8.258	1.287	0.153	0.0368	0.00908
$\delta_0/\%$ (diff)	-7.3	-7.3	-7.3	-7.3	-7.3	-7.3	-7.3
$\hat{\sigma}_1/\text{pb}, \overline{\text{MS}}(M_W), [G_F]$	2.665	8183.2	7.796	1.345	0.183	0.0467	0.01195
$\hat{\sigma}_1/\text{pb}, \overline{\text{MS}}(M_W), [\alpha(0)]$	2.617	8029.5	7.721	1.324	0.179	0.0455	0.01162
$\delta_1/\%$ (diff)	-1.8	-2.0	-0.5	-1.5	-2.6	-3.1	-3.3

Table 3  
The total parton cross section in the  $G_F$  and  $\alpha(0)$  EW schemes.

the quark masses with and without the subtraction procedure. The conditions are as follows: the center-of-mass energy is 200 GeV, all events with the invariant mass of the neutrino and charged lepton pair above  $\sqrt{40}$  GeV are accepted.  $\sigma_0$  denotes the Born-level cross section obtained using the CTEQ4L set of PDF's [23].  $\sigma_1$ ,  $\sigma_1^{\overline{\text{MS}}(\sigma)}$ ,  $\sigma_1^{\overline{\text{MS}}(q)}$ , and  $\sigma_1^{\overline{\text{MS}}(q)}(\text{lin.})$  stand for the cross sections with one-loop EW RC included. The double counting of the quark mass singularities in  $\sigma_1$  is not removed. The  $\overline{\text{MS}}$  procedure (12) is applied to the partonic cross section in the computation of  $\sigma_1^{\overline{\text{MS}}(\sigma)}$ . Values of  $\sigma_1^{\overline{\text{MS}}(q)}$  and  $\sigma_1^{\overline{\text{MS}}(q)}(\text{lin.})$  are computed by convolution of the quark (parton) density function modified according to Eq. (13) with the full (including quark mass singularities) partonic cross section. The linearization procedure (14) was adopted for  $\sigma_1^{\overline{\text{MS}}(q)}(\text{lin.})$  in addition. One can see that the numerical effect of linearization is small (but visible) at least for the considered set-up. The two approaches to remove the double counting give very close results as it should be.

	$\sigma_0$ [pb]	$\sigma_1$	$\sigma_1^{\overline{\text{MS}}(\sigma)}$	$\sigma_1^{\overline{\text{MS}}(q)}$	$\sigma_1^{\overline{\text{MS}}(q)}(\text{lin.})$
$m_u = m_d = 4.85$ MeV	2.5577(1)	2.4795(3)	2.5724(3)	2.5704(3)	2.5729(3)
$m_u = m_d = 48.5$ MeV	2.5577(1)	2.4992(3)	2.5724(3)	2.5713(3)	2.5727(3)
$m_u = m_d = 485.$ MeV	2.5577(1)	2.5190(3)	2.5724(3)	2.5719(3)	2.5726(3)

Table 4  
The tree level and corrected hadronic Drell–Yan cross section different values of the light quark masses.

In this way with help of the automatized SANC system we calculated the complete one-loop radiative corrections to the charged current Drell-Yan cross section. Our results at the partonic level are in fair agreement with the ones published earlier in Ref. [5]. The corresponding computer codes in analytical (FORM) and numerical (FORTRAN) formats are available from SANC [8]. They can be used as a part of a more general computer program (like a Monte Carlo event generator) to describe the Drell-Yan process in realistic conditions. Further comparisons at the hadronic level are in progress.

## Acknowledgements

We are grateful to S. Dittmaier, S. Jadach, W. Placzek, Z. Was for discussions. This work was supported by the INTAS grant 03-51-4007. Three of us (D.B., L.K. and G.N.) are indebted to the directorate of IFJ, (Cracow, Poland) for a hospitality extended to them in April–May 2005, when a part of this study was completed. One of us (A.A.) thanks the RFBR grant 04-02-17192.

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