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Atomic Parity-Violation and Precision Physics

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Abstract

The atomic parity-violation (APV) parameter Q_W for a nucleus with n neutrons and z protons has been included in the list of pseudo-observables accessible with the codes `TOPAZ0` and `ZFITTER`. In this way one can add the APV results in the LEP EWWG 'global' electroweak fits, checking the corresponding effect when added to the existing precision measurements.

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1 Introduction

Recently we have been asked to include atomic-parity violation (hereafter APV) parameters in the list of pseudo-observables (hereafter PO) that are accessible with the `FORTRAN` codes `TOPAZO` [1] and `ZFITTER` [2], so to include the APV results in the LEP EWWG electroweak fits.

The reason for this operation is that there are now precise experiments measuring APV in cesium [3], at the 0.4% level, thallium [4], lead [5] and bismuth [6]. Moreover, according to [7], the uncertainties associated with the atomic wave-functions have been reduced to another 0.4% for cesium. For additional uncertainties associated with the value of the tensor polarizability we refer to [8]. Note however that there is an intrinsic difference between the PO at the Z-resonance, e.g. Γ_{Z}, σ^0_h, $A^0_{\rm FB}$ etc, and the APV parameters where the typical scale is dictated by the limit of zero momentum transfer in the APV Hamiltonian. This fact alone is the origin of a comparatively larger theoretical uncertainty which is due to our basic ignorance of QCD corrections in this regime.

The investigation of APV has been the subject of a number of studies made in the 80's by Marciano and Sirlin [9] and [10]. For `TOPAZO`, which is based on the generalized minimal subtraction scheme [11], it has been relatively simple to include all recently computed higher-order effects in the old $\overline{MS}$ calculation. For `ZFITTER` instead, the authors have been able to produce a novel evaluation of the APV parameters in the on mass-shell (OMS) scheme. The current value for the weak-charge is

$$Q_W({\rm Cs}) = -72.06 \pm 0.28 \pm 0.34\,({\rm theo.}) \tag{1}$$

For a recent evaluation of Q_W we refer, again, to [8] where the program `GAPP` [12] has been used.

2 Upgrading the $\overline{MS}$ calculation

The electron–quark parity-violating Hamiltonian at zero momentum-transfer will be conventionally parametrized as follows:

$$H_{PV} = \frac{G_F}{\sqrt{2}}\Big(C_{1u}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma_\mu u + C_{2u}\bar{e}\gamma_\mu e\bar{u}\gamma_\mu\gamma_5 u + C_{1d}\bar{e}\gamma_\mu\gamma_5 e\bar{d}\gamma_\mu d + C_{2d}\bar{e}\gamma_\mu e\bar{d}\gamma_\mu\gamma_5 d\Big), \tag{2}$$

where the ellipsis represents heavy-quark terms and we have factorized out the Fermi constant G_F. In heavy atoms the dominant part of parity-violation is proportional to the so-called weak-charge Q_W

$$Q_W(Z,A) = 2\Big[(Z+A)\;C_{1u} + (2\,A - Z)\;C_{1d}\Big], \tag{3}$$

We have taken the calculation by Marciano and Sirlin which is performed in the modified minimal subtraction scheme ($\overline{MS}$) and have extended it to include all higher-order effects presently known. To summarize: two-loop leading contribution for the ρ-parameter [13], exact $\mathcal{O}(\alpha\alpha_S)$ corrections [14], $\mathcal{O}\left(\alpha\alpha_S^2\right)$ corrections to ρ [15], next-to-leading two-loop heavy top corrections [16]. At the same time, an attempt has been made to evaluate the theoretical uncertainty at the level of electroweak and of QCD corrections.

`TOPAZO` now returns, among all PO, the two quantities, C_{1u} and C_{1d} of Eq.(2). They are defined as follows:

$$\begin{aligned} C_{1u} &= -\frac{1}{2}\,\rho'_{\rm PV}\left[1 - \frac{8}{3}\,\kappa'_{\rm PV}(0)\sin^2\hat{\theta}(M^2_W)\right], \\ C_{1d} &= \frac{1}{2}\,\rho'_{\rm PV}\left[1 - \frac{4}{3}\,\kappa'_{\rm PV}(0)\sin^2\hat{\theta}(M^2_W)\right], \end{aligned} \tag{4}$$

where $\sin^2\hat{\theta}(M_W^2)$ is the $\overline{MS}$ weak-mixing angle at the scale $\mu = M_W$. We adopt a specific implementation of the re-summation procedure where the pair M_W and $\sin^2\hat{\theta}(M_W^2)$ is the solution of a system of coupled non-linear equations that include all available higher-order effects, as described in Sect. 6.11 and 8 of ref. [17]. Moreover,

$$\begin{aligned}
\rho'_{\rm PV} &= \rho - \frac{\alpha}{2\pi}\left[1+\frac{1}{\hat{s}^2}+4\,\hat{v}_e\,B_{\rm p(np)}+\frac{9}{16\,\hat{s}^2\hat{c}^2}\left(1-\frac{16}{9}\hat{s}^2\right)\left(1+\hat{v}_e^2\right)\right],\\
\kappa'_{\rm PV}(0) &= \kappa_{\rm PV}(0) - \frac{\alpha}{2\pi\hat{s}^2}\left[\frac{9-8\hat{s}^2}{8\,\hat{s}^2}-\frac{\hat{v}_e}{6}\left(\ln\frac{M_Z^2}{m_e^2}+\frac{1}{6}\right)+\left(\frac{9}{4}-4\hat{s}^2\right)\hat{v}_e\,B_{\rm p(np)}\right.\\
&\left.+\frac{9}{16\,\hat{s}^2\hat{c}^2}\left(\frac{1}{2}\hat{v}_e+\frac{16}{9}\hat{s}^4\right)\left(1+\hat{v}_e^2\right)\right], \qquad (5)
\end{aligned}$$

where $\hat{v}_e = 1-4\,\hat{s}^2$, $\hat{s}^2 = \sin^2\hat{\theta}(M_W^2)$ and where we have two different treatments of the Z–γ boxes — perturbative [9]

$$B_{\rm p} = \ln\frac{M_Z^2}{m^2}+\frac{3}{2}, \qquad m = m_u = m_d = 75\,{\rm MeV}, \qquad (6)$$

and non-perturbative [10]

$$\begin{aligned}
B_{\rm np} &= K + \frac{4}{5}\,(\xi_1)_B^p\,,\\
K &= M_Z^2\int_{M^2}^{\infty}\frac{du}{u(u+M_Z^2)}\left[1-\frac{\alpha_S(u)}{\pi}\right], \qquad (\xi_1)_B^p = 2.55, \qquad (7)
\end{aligned}$$

where M is a mass scale representing the onset of the asymptotic behavior, i.e. the regime where α_S becomes small. We observe a plateau of stability in K for M centered around 0.5 GeV and this is the numerical value used. Furthermore we used the following form for the ρ, κ parameters [9]:

$$\begin{aligned}
\rho &= 1+\frac{\alpha}{4\pi\hat{s}^2}\left[\frac{3}{4\,\hat{s}^2}\ln\hat{c}^2-\frac{7}{4}+\frac{3}{4}\frac{m_t^2}{M_W^2}\left(1+\delta_{\rm EW}+\delta_{\rm QCD}\right)\right.\\
&\left.+\frac{3}{4}h\left(\frac{\ln(\hat{c}^2/h)}{\hat{c}^2-h}+\frac{1}{\hat{c}^2}\frac{\ln h}{1-h}\right)\right],\\
\kappa_{\rm PV}(0) &= 1-\frac{\alpha}{2\pi\hat{s}^2}\left[\frac{7}{9}-\frac{\hat{s}^2}{3}+\frac{Q_f}{3}\sum_f\left(I_e^{(3)}-2\,Q_f\hat{s}^2\right)\ln\frac{m_f^2}{M_W^2}\right]. \qquad (8)
\end{aligned}$$

In the previous equation we have $h = M_H^2/M_Z^2$. The strange quark mass is effectively chosen to be $m_s = 250\,{\rm MeV}$ so that (with effective $m_u = m_d = 75\,{\rm MeV}$) we recover the dispersive analysis for the Z–γ transition where $\Pi_{Z\gamma}(0)$ is rewritten in terms of a dispersion relation with the kernel connected to $\sigma(e^+e^- \to \text{hadrons})$. Inside Eq.(8) $\delta_{\rm EW(QCD)}$ are the LO+NLO electroweak $\left(\mathcal{O}\left(\alpha_S^2+\alpha_S^3\right)\ {\rm QCD}\right)$ correction to ρ. The evaluation of ρ and $\hat{s}^2$ includes the best available LO+NLO terms [17].

`TOPAZ0` default is the perturbative formulation of the *factorized* result of Eq.(4). There is the option of using some *additive* formulation where

$$\begin{aligned}
C_{1u} &= -\frac{1}{2}\rho\left[1-\frac{8}{3}\kappa_{\rm PV}(0)\sin^2\hat{\theta}(M_W^2)\right]+\Delta_u\,,\\
C_{1d} &= \frac{1}{2}\rho\left[1-\frac{4}{3}\kappa_{\rm PV}(0)\sin^2\hat{\theta}(M_W^2)\right]+\Delta_d\,, \qquad (9)
\end{aligned}$$

where $\Delta_{u,d}$ are obtained from Eq.(4) by expanding and by neglecting terms of $\mathcal{O}(\alpha^2)$.

3 Atomic parity-violation in OMS scheme

The old result of [9] has been completely re-derived in the OMS scheme. Here, the technical problem is represented by the extraction of the limit of zero momentum transfer from the expressions that have been derived for the process $ee \to t\bar{t}$ [18]. Here the process under consideration is the t-channel scattering $ee \to uu$ and what we need is naturally contained in the results of Ref. [18] since they were derived retaining all masses and, therefore, the limit of zero momentum transfer, $Q^2 << (\text{all})\ m^2$ is possible.

It is rather easy to take the limit $Q^2 \to 0$ for vertices and self-energy functions since they depend only on this variable. For boxes the procedure is more complex due to their complicated dependence on s and t invariants. Fortunately enough, ZZ and $Z\gamma$ boxes form a gauge invariant sub-set of the whole result and for WW boxes one has to replace, in the corresponding limit, only the $\xi = 1$ part of the result, which is well defined and simple. This fact triggered the strategy for a calculation where we take all contributions but boxes from the $Q^2 \to 0$ limit of the $ee \to t\bar{t}$ form factors and were we have re-computed, from scratch, box diagrams at $Q^2 = 0$. Note that this calculation was done with the aid of the computer system described in Ref. [19].

For our calculation we compare the APV Hamiltonian of Eq.(2) with its $ee \to t\bar{t}$ analog, Eq.(I.10) of [18]:

$$\begin{aligned}\mathcal{A}_Z(0) &= I_e^{(3)} I_f^{(3)} \frac{\pi\alpha}{s_W^2 c_W^2 (-M_Z^2)} \Big\{ \gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_+ F_{LL}(0) + d_e \gamma_\mu \otimes \gamma_\mu\gamma_+ F_{QL}(0) \\ &\quad + d_f \gamma_\mu\gamma_+ \otimes \gamma_\mu F_{LQ}(0) + d_e d_f \gamma_\mu \otimes \gamma_\mu F_{QQ}(0) \Big\}. \end{aligned} \tag{10}$$

Here (0) stands for $Q^2 = 0$ and we write only one argument since box contributions are excluded. Moreover,

$$\gamma_+ = 1 + \gamma_5\,, \qquad d_f = -4|Q_f| s_W^2\,. \tag{11}$$

From Eqs.(2) and (10) we immediately derive a relation between the APV parameters C_{1f} and C_{2f} and the $ee \to t\bar{t}$ form factors at zero momentum transfer:

$$\begin{aligned} C_{1f} &= I_f^{(3)} \Big[f_{LL} + d_f f_{LQ} - \Delta r\,(1 + d_f) \Big], \\ C_{2f} &= I_f^{(3)} \Big[f_{LL} + d_e f_{QL} - \Delta r\,(1 + d_e) \Big]. \end{aligned} \tag{12}$$

Here $f = u, d$ and

$$f_{LL,QL,LQ} = 1 + \frac{\alpha}{4\pi s_W^2} F_{LL,QL,LQ}(0). \tag{13}$$

After a lengthy but straightforward calculations, we are able to reproduce the following generic expressions:

$$\begin{aligned} C_{1u} &= -2 I_e^{(3)} \rho_{PV} \left(I_u^{(3)} - 2 Q_u \kappa_{PV} s_W^2 \right) \\ &\quad + \frac{\alpha}{\pi} \Bigg[Q_e^2 a_e v_u + \frac{1}{3} Q_u Q_\nu \left(\ln r_{We} + \frac{1}{6} \right) + \frac{2}{3} Q_u Q_e v_e a_e \left(\ln r_{Ze} + \frac{1}{6} \right) \\ &\quad + C_f^{WW} + 3 Q_u a_u Q_e v_e \left(\ln r_{Zu} + \frac{3}{2} \right) + \frac{3}{4 s_W^2 c_W^2} v_u a_u \left(v_e^2 + a_e^2 \right) \Bigg], \end{aligned} \tag{14}$$

where we introduced a notation $r_{ij} = m_i^2/m_j^2$ and a fictitious term with non-zero neutrino charge in order to have a completely general representation and where

$$C_f^{WW} = \begin{cases} \dfrac{1}{2s_W^2} & \text{for } f = u\,, \\ -\dfrac{1}{8s_W^2} & \text{for } f = d\,, \end{cases} \tag{15}$$

is a contribution, originating from the WW box, which is different for u and d channels (direct–crossed).

The other APV parameters can be obtained with the aid of some simple substitutions:

$$C_{2u} = C_{1u}\Big|_{e \leftrightarrow u,\, Q_\nu \to Q_d}, \qquad C_{1(2)d} = C_{1(2)u}\Big|_{u \leftrightarrow d}. \tag{16}$$

The terms of the second and third rows of Eq.(14) are identical to corresponding terms of the $\overline{MS}$ result. In the sequential order they are due to: QED vertex in Z exchange, W abelian vertex in γ exchange (with neutrino charge), Z abelian vertex in γ exchange; WW, $Z\gamma$ and ZZ boxes.

The only difference with respect to the $\overline{MS}$ result is present in the first term. The factor ρ is almost the same:

$$\rho_{PV} = 1 + \frac{\alpha}{4\pi s_W^2}\left\{\frac{3}{4}\left[-\frac{1}{s_W^2}\ln c_W^2 - \frac{r_{HW}}{1-r_{HW}}\ln r_{HW} + \frac{r_{HW}}{1-r_{HZ}}\ln r_{HZ}\right] - \frac{7}{4} - \Delta\rho^{\text{fer}}(0)\right\}, \tag{17}$$

where we use instead the full expression for $\rho^{\text{fer}}(0)$,

$$\Delta\rho^{\text{fer}}(0) = \frac{\Sigma^{\text{fer}}_{WW}(0) - \Sigma^{\text{fer}}_{ZZ}(0)}{M_W^2}\,, \tag{18}$$

contrary to the approximation made above where only the (leading) quadratic term in m_t is retained. The difference, being proportional to light fermion masses is numerically rather small.

However, the main difference with the $\overline{MS}$ calculations is confined in the APV parameter κ_{PV} for which, in the OMS-scheme, we derived:

$$\kappa_{PV} = 1 + \frac{\alpha}{4\pi s_W^2}\left\{\left(\frac{1}{6} + 7c_W^2\right)L_\mu(M_W^2) - \frac{8}{9} - \frac{2}{3}c_W^2 - \frac{c_W^2}{s_W^2}\left(\Delta\rho^{\text{bos},F} + \Delta\rho^{\text{fer},F}\right) - \Pi^{\text{fer}}_{Z\gamma}(0)\right\}, \tag{19}$$

where $L_\mu(M_W^2) = \ln\dfrac{M_W^2}{\mu^2}$. The gauge invariant Veltman $\Delta\rho$ parameter is

$$\Delta\rho = \frac{\Sigma^{\text{fer}}_{WW}(M_W^2) - \Sigma^{\text{fer}}_{ZZ}(M_Z^2)}{M_W^2}\,, \tag{20}$$

and contains both the *bosonic* and the *fermionic* components. We explicitly give the bosonic part, $\Delta\rho^{\text{bos}}$ (for definition of finite part B_0^F of B_0 functions see [17]):

$$\begin{aligned} \Delta\rho^{\text{bos},F} \;=\; & \left(\frac{1}{12c_W^4} + \frac{4}{3c_W^2} - \frac{17}{3} - 4c_W^2\right)\left[B_0^F\left(-M_W^2; M_W, M_Z\right) - c_W^2 B_0^F\left(-M_Z^2; M_W, M_W\right)\right] \\ & + \left(1 - \frac{1}{3}r_{HW} + \frac{1}{12}r_{HW}^2\right)B_0^F\left(-M_W^2; M_W, M_H\right) \\ & - \left(1 - \frac{1}{3}r_{HZ} + \frac{1}{12}r_{HZ}^2\right)\frac{1}{c_W^2}B_0^F\left(-M_Z^2; M_Z, M_H\right) - 4s_W^2 B_0^F\left(-M_W^2; M_W, 0\right) \\ & + \frac{1}{12}\left[\left(\frac{1}{c_W^4} + \frac{6}{c_W^2} - 24 + r_{HW}\right)L_\mu(M_Z^2) + s_W^2 r_{HW}^2\left[L_\mu(M_H^2) - 1\right]\right. \\ & \left. - \left(\frac{1}{c_W^2} + 14 + 16c_W^2 - 48c_W^4 + r_{HW}\right)L_\mu(M_W^2) - \frac{1}{c_W^4} - \frac{19}{3c_W^2} + \frac{22}{3}\right]. \end{aligned} \tag{21}$$

To establish a link with the $\overline{MS}$ calculation we introduce the usual notion of leading and reminder terms:

$$\kappa_{PV} \;=\; 1+\frac{\alpha}{4\pi s^2_{W}}\Bigl\{(\Delta\kappa_{PV})_{\rm lead}+(\Delta\kappa_{PV})_{\rm rem}\Bigr\}\,, \tag{22}$$

where the leading term contains only $\Delta\rho$ and the reminder contains all the rest:

$$\begin{aligned}(\Delta\kappa_{PV})_{\rm lead} &= -\frac{c^2_{W}}{s^2_{W}}\left(\Delta\rho^{{\rm bos},F}+\Delta\rho^{{\rm fer},F}\right)\Big|_{\mu=M_W}\,,\\ (\Delta\kappa_{PV})_{\rm rem} &= -\frac{8}{9}-\frac{2}{3}c^2_{W}-\Pi^{\rm fer}_{Z\gamma}(0)\Big|_{\mu=M_W}\,.\end{aligned} \tag{23}$$

Numerically, $(\Delta\kappa_{PV})_{\rm lead}$ and $(\Delta\kappa_{PV})_{\rm rem}$ are nearly equal and one might think that the usual leading–reminder splitting, the standard factorization of contributions with different scales and re-summation (see [17]),

$$\kappa_{PV}=\left[1+\frac{\alpha}{4\pi s^2_{W}}\left(\Delta\kappa_{PV}\right)_{\rm lead}\right]\left[1+f_c\frac{\alpha}{4\pi s^2_{W}}\left(\Delta\kappa_{PV}\right)_{\rm rem}\right], \tag{24}$$

with a conversion factor

$$f_c=\frac{\sqrt{2}G_\mu M^2_{Z}s^2_{W}c^2_{W}}{\pi\alpha}\,, \tag{25}$$

is not too well justified for the APV parameter κ_{PV}. Note, however, that the factorized form of Eq.(24) is fully consistent with the $\overline{MS}$ result Eq.(8) if we identify

$$\sin^2\hat{\theta}_W(M_W)=\left[1-f_c\frac{\alpha}{4\pi}\frac{c^2_{W}}{s^4_{W}}\;\Delta\rho^F\Big|_{\mu=M_W}\right]s^2_{W}\,. \tag{26}$$

As done before, for $\Pi^{\rm fer}_{Z\gamma}(0)$ we use effective quark masses which are consistent with a dispersive treatment of $\Pi^{\rm fer}_{Z\gamma}$ at zero scale.

Finally, we apply mixed QCD $\left(\mathcal{O}\left(\alpha^2_{S}+\alpha^3_{S}\right)\right)$ and LO+NLO electroweak two-loop corrections for Veltman $\Delta\rho$ parameter. For ρ_{PV} we stick with one-loop (non-re-summed) result Eq.(17), since there the notion of leading–reminder splitting fails completely (numerically it looks like $+3.5-3.0=0.5$). For the same reason, we apply to $\Delta\rho^{\rm fer}(0)$ only the mixed QCD but not the electroweak two-loop corrections, as already done in the first Eq.(8). The latter, as well as the other electroweak NLO corrections for remainder terms, although not implemented are successively used to evaluate the theoretical uncertainty in the electroweak sector of the OMS scheme.

4 Theoretical uncertainty in APV

In order to discuss the present level of theoretical uncertainty in atomic parity-violation we start with the $\overline{MS}$ results for $Q_W(\mathrm{Cs})$ that are shown in Tab. 1, corresponding to $M_Z=91.1875\,\mathrm{GeV}$, $M_H=150\,\mathrm{GeV}$ and $\alpha_S(M^2_Z)=0.119$. `ZFITTER` numbers corresponding to 'Add/Pert' setup are added to the third row of the Table. The m_t-dependence of $Q_W(\mathrm{Cs})$ is shown in Tab. 1 where we register a 0.22(0.17) per-mill increase for m_t between 170 GeV and 180 GeV and for Pert(Non Pert). As for the M_H dependence we have computed a decrease of about 0.7 per-mill for M_H between 150 GeV and 300 GeV.

m_t [GeV]	170	175	180
Fact/Pert	-72.9712	-72.9632	-72.9551
Fact/Non Pert	-73.1994	-73.1932	-73.1869
Add/Pert	-72.9732	-72.9658	-72.9582
ZFITTER	-72.9762	-72.9698	-72.9637
Add/Non Pert	-73.2026	-73.1969	-73.1912

Table 1: Predictions for $Q_W(\mathrm{Cs})$ from TOPAZ0 and ZFITTER for $M_Z = 91.1875\,\mathrm{GeV}$, $M_H = 150\,\mathrm{GeV}$ and $\alpha_S(M_Z^2) = 0.119$.

The associated theoretical uncertainty is approximately 3.2 per-mill and it is largely dominated by QCD effects. Let us consider the main sources of uncertainty. In the original calculation of Marciano and Sirlin we have a dependence of the result on light quark masses. This appearance can be seen in Eq.(5) and in the perturbative treatment of boxes, Eq.(6).

From 1983 the accuracy associated to the weak-charge Q_W has been considerably reduced and we cannot include it in the list of high-precision PO if the result contains logarithmic enhancements due to light quark masses.

In their second paper Marciano and Sirlin [10] have suggested how to go beyond the partonic-language. One should distinguish quark masses in the Z–γ transition and in Z–γ boxes. Light quark masses, m_{uds}, are then fixed to parameterize the dispersive result for the Z–γ transition and are not varied anymore in evaluating the theoretical uncertainty.

Furthermore we have Z–γ box diagrams where quark masses show up as the consequence of the zero momentum transfer limit. Here, according to the suggestion of [10] we split the boxes into a low-frequency part, approximated with the Born contribution for a physical nucleon (the $(\xi_1)^p_B$ term in Eq.(7)), and an high-frequency part (the K-term in Eq.(7)) that includes $\mathcal{O}(\alpha_S)$ corrections where light quark masses disappear. The mass scale M separating low- from high-frequency parts is, of course, arbitrary and only subjected to the requirement that $\alpha_S(Q^2)$ starts to become small for $\mid Q^2 \mid > M^2$ and that $M > \Lambda_{\mathrm{QCD}}$. However, with the most complete evaluation of α_S (up to three loops) we have found a plateau of stability for the result, i.e. for M between 0.5 and 0.6(0.8) K goes from 9.2016 to 9.1737(8.7818) and Q_W has a variation of 0.02(0.3) per-mill. Therefore we fix $0.5 \leq M \leq 0.6$.

Instead of varying light quark masses between undefined limits we prefer to estimate the theoretical uncertainty by comparing the perturbative result with light quark masses fixed to reproduce the dispersive approach to the Z–γ transition with a non-perturbative anzatz based on a low-frequency high frequency splitting at a mass scale of about 0.5 GeV. Note that when comparing $B_{\mathrm{np}}(M)$ with the perturbative factor $\ln M_Z^2/M^2 + \frac{3}{2}$ we find that the perturbative approach overestimates the effect of about 5.6%(1.9%) at $M = 0.5(0.8)\,\mathrm{GeV}$.

Furthermore, the differences in the factorized Eq.(4) versus additive Eq.(9) formulation of the coefficients $C_{1u,1d}$ is approximately 0.05 per-mill signaling that, from TOPAZ0's treatment alone, pure electroweak higher orders are relatively under control. Another way of testing the electroweak theoretical uncertainty is, as usual, to compare two different renormalization schemes with the same input parameter set. When we compare ZFITTER in the preferred setup with TOPAZ0 additive/perturbative we obtain a relative difference of 0.04(0.05, 0.08) per-mill at $m_t = 170(175, 180)\,\mathrm{GeV}$. However, an internal evaluation of electroweak theoretical uncertainties within ZFITTER (realized by evaluating the effect of the electroweak two-loop corrections which are not included in the preferred setup) shows a value of about ± 0.25 per-mill. Again, the conclusion is that theoretical uncertainty is completely dominated by QCD effects at zero momentum transfer.

Finally, let us define an effective APV weak-mixing angle by the following relation:

$$\sin^2\theta_{\rm APV} = \kappa'_{\rm PV}(0)\sin^2\hat\theta(M_{\scriptscriptstyle W}^2). \tag{27}$$

For $M_{\scriptscriptstyle Z} = 91.1875\,{\rm GeV}$, $M_{\scriptscriptstyle H} = 150\,{\rm GeV}$ and $\alpha_{\scriptscriptstyle S}(M_{\scriptscriptstyle Z}^2) = 0.119$ we obtain $\sin^2\theta_{\rm APV} = 0.231601$ (0.232123), corresponding to perturbative (non-perturbative) treatment.

Appendix: Taylor expansions Here we list all expansions that are needed in order to reproduce the OMS results. Note that we need at most terms of $\mathcal{O}(s)$:

$$C_0\left(-m_u^2,-m_u^2,-s;m_u,M,m_u\right) = \frac{1}{M^2}\Biggl\{-1-\frac{7}{2}r_{uM}-\frac{37}{3}r_{uM}^2-\left(1+3r_{uM}+10r_{uM}^2\right)\ln r_{uM}$$
$$+\frac{s}{M^2}\left[\frac{1}{6r_{uM}}+\frac{13}{12}+\frac{52}{9}r_{uM}+\frac{673}{24}r_{uM}^2+\left(\frac{1}{2}+\frac{10}{3}r_{uM}+\frac{35}{2}r_{uM}^2\right)\ln r_{uM}\right]\Biggr\}, \tag{28}$$

where $M = M_{\scriptscriptstyle Z}, M_{\scriptscriptstyle H}$ and we remind a short hand notation for mass ratios: $r_{ij} = \dfrac{m_i^2}{m_j^2}$.

The other expansions read:

$$C_0\left(-m_u^2,-m_u^2,-s;m_d,M_{\scriptscriptstyle W},m_d\right) = \frac{1}{M_{\scriptscriptstyle W}^2}\Biggl\{-1-r_{dW}-r_{dW}^2-\left(\frac{5}{2}+8r_{dW}\right)r_{uW}-\frac{10}{3}r_{uW}^2$$
$$-\left[1+2r_{dW}+3r_{dW}^2+(1+6r_{dW})\,r_{uW}+r_{uW}^2\right]\ln r_{dW}$$
$$+\frac{s}{M_{\scriptscriptstyle W}^2}\Biggl(\frac{1}{6}\left[\frac{1}{r_{dW}}+\frac{11}{2}+13r_{dW}+\frac{47}{2}r_{dW}^2+\left(\frac{1}{r_{dW}}+\frac{62}{3}+97r_{dW}\right)r_{uW}+\left(\frac{1}{r_{dW}}+\frac{187}{4}\right)r_{uW}^2\right.$$
$$\left.+\frac{1}{r_{dW}}r_{uW}^3\right]+\left[\frac{1}{2}+2r_{dW}+5r_{dW}^2+\left(\frac{4}{3}+10r_{dW}\right)r_{uW}+\frac{5}{2}r_{uW}^2\right]\ln r_{dW}\Biggr)\Biggr\}, \tag{29}$$

$$C_0\left(-m_u^2,-m_u^2,-s;M_{\scriptscriptstyle W},0,M_{\scriptscriptstyle W}\right) = \frac{1}{M_{\scriptscriptstyle W}^2}\left[1+\frac{r_{uW}}{2}+\frac{r_{uW}^2}{3}+\frac{s}{6M_{\scriptscriptstyle W}^2}\left(\frac{1}{2}+\frac{1}{3}r_{uW}\right)\right], \tag{30}$$

$$C_0\left(0,0,0;M_{\scriptscriptstyle H},0,M_{\scriptscriptstyle Z}\right) = -\frac{1}{M_{\scriptscriptstyle Z}^2-M_{\scriptscriptstyle H}^2}\ln r_{HZ}\,, \tag{31}$$

$$B_0^F\left(-m_u^2;M_{\scriptscriptstyle W},0\right) = -L_\mu(M_{\scriptscriptstyle W}^2)+1+\frac{1}{2}r_{uW}+\frac{1}{6}r_{uW}^2\,, \tag{32}$$

$$B_0^F\left(-m_u^2;M,m_u\right) = -L_\mu(M^2)+\left(r_{uM}+2r_{uM}^2\right)\ln r_{uM}+1+\frac{1}{2}r_{uM}+\frac{5}{3}r_{uM}^2\,, \tag{33}$$

$$\begin{aligned} B_{0p}\left(-m_u^2;M,m_u\right) &= -\frac{1}{2M^2}\,,\\ B_0^F\left(-s;m,m\right) &= -L_\mu(m^2)+\frac{s}{6m^2}\,, \end{aligned} \tag{34}$$

$$\begin{aligned} B_0^F\left(-s;m,M\right) &= 1+\left[M^2L_\mu(M^2)-m^2L_\mu(m^2)\right]\frac{1}{m^2-M^2}\\ &\quad +s\left[\frac{m^2+M^2}{2(m^2-M^2)^2}-\frac{m^2M^2}{(m^2-M^2)^3}\ln\left(\frac{m^2}{M^2}\right)\right]. \end{aligned} \tag{35}$$

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