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# 12 Years of Precision Calculations for LEP. What's Next?

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## Abstract

I shortly review time period of twelve years, 1989-2000, which was devoted to a theoretical support of experiments at LEP and SLC at  $Z$  resonance and discuss several directions of possible future work in the field of precision theoretical calculations for experiments at future colliders.

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## 1 Introduction

Exiting 50 years of foundation of Precision EW Physics were perfectly reviewed in several brilliant talks at this Symposium [1]. I hope, our community would share my opinion that after 12 years of excellent work of LEP we have full rights to say that a new scientific discipline has been born, Precision High-Energy Physics, **PHEP**, consisting of:

- experimental measurements themselves at per mill precision level;
- supporting theoretical calculations with even better precision.

Me and my colleagues were involved into this glorious period with ZFITTER project. Actually, ZFITTER project has been started 25 years ago, in 1976, when we wrote our papers on *on-mass-shell renormalization scheme*. Formally, first ZFITTER was born within CERN workshop “Z Physics at LEP 1” in 1989. As an important intermediate step I would like to mention CERN workshop 1994–1995, “Precision calculations for Z resonance”, which I had pleasure to run and to have Alberto among our contributors. In 1999 we have published a long write-up of ZFITTER v6.21, which recently appeared in CPC [2]. When I say “12 Years of Precision Calculations for LEP”, I mean that during all lifetime of LEP, **1989–2000**, ZFITTER was upgraded and supported in ADLO, SLC and LEPEWWG. We would like to see **the end** of ZFITTER project in the Y2K and, therefore, a very natural question arises: **What's next?**

Meantime, me and Giampiero Passarino in **1997-1998** wrote a monstrous book: “The Standard Model in the Making”, which appeared in 1999 in Oxford University Press [3]. It has some value for this talk.

ZFITTER uses an one-loop core and general environment based on our own formulae, and incorporates all the world results for higher order QED, QCD and EW Radiative Corrections (EWRC). As an input it uses the so-called **LEP1, IPS** – Input Parameter Set, i.e. 5 parameters:

$$\Delta\alpha_h^{(5)}(M_Z^2), \quad \alpha_s(M_Z^2), \quad m_t, \quad M_Z, \quad \mathbf{M}_H. \quad (1)$$

$M_Z$  is measured very precise at LEP1 and for the first three parameters a rich information is available from the other measurements. Therefore, we are

approaching one-parameter fit with  $M_H$  being the only parameter. Results of such a fit are being presented in famous **Blue Band** figure, derived with the aid of TOPAZO & ZFITTER codes.

ZFITTER incorporates all known in the literature QED RC up to  $\mathcal{O}((\alpha L)^3)$  (where  $L = \ln s/m_e^2 - 1 = 23$  at  $s = M_Z^2$  and, therefore, effective QED coupling is quite large:  $\alpha L = 0.169$ ); QCD RC up to  $\mathcal{O}(\alpha_s^3)$ ; and EWRC up to  $\mathcal{O}(\alpha^2)$ , however, for the latter only *leading* and *subleading* RCs are known (*Degrassi, Gambino, Sirlin et al., 1996–1997*).

In several cases perturbative calculations are *saturated* like it takes place for leptonic contribution to the running QED coupling,  $\Delta\alpha_l(s)$ , which is presently known up to three loops (2-loops *Källén, 1955*; 3-loops *Steinhauser, 1998*):

$$\Delta\alpha_l = 314.97637 \cdot 10^{-4} = [314.18942_{1\text{-loop}} + 0.77616_{2\text{-loop}} + 0.01079_{3\text{-loop}}] \cdot 10^{-4}. \quad (2)$$

The last known term is small enough to be used as an *estimator* of theoretical uncertainty. Typical scales of the problem: LEP1,2 energies  $\sqrt{s} = M_Z - 200$  GeV,  $M_W, M_Z, m_t, M_H$ , all are of the same order and calculations must be **complete** and one-loop calculations were complete! For two-loops, the notion of  $m_t^2$  *enhanced terms* was introduced. Only terms  $\mathcal{O}(G_F m_t^4)$  and  $\mathcal{O}(G_F m_t^2 M_Z^2)$  are known. Since likely,  $100 \text{ GeV} \leq M_H \leq 250 \text{ GeV}$ , popular expansions in  $M_H^2/m_t^2$  and  $m_t^2/M_H^2$  have bad convergence. One may say that **complete two-loop EWRC were welcome for LEP1 but our community did not deliver them!**

In this talk I will tell about three directions of possible future work in which we were digging in the Y2K. The first one we call:

## 2 Book “heritage”

While working on the book, me and Giampiero Passarino wrote dozens of book supporting form codes. Later on *an idea was erupted to collect, order, unify and upgrade these codes up to the level of a “computer system”*: to which we gave the name CalcPHEP — *’Calculus of Precision High Energy Physics*.

This system is being realized at a web site: `brg.jinr.ru`. Presently we do not maintain any author list, we say that we have people who contributed to it: *G.Passarino, DB, L.Kalinovskaya, P.Christova, G.Nanova and A.Andonov* [4].

After completion of R&D phase of the project, presently the following options are available at the site:

1. generation and reduction to the scalar PV functions of one-loop Feynman diagrams (FD) for **all** SM  $1 \rightarrow 2$  decays and  $2f \rightarrow 2f$  processes (in  $R_\xi$  gauge, QCD included);
2. computation of one-loop scalar form factors for decays  $Z(H) \rightarrow f\bar{f}$  and  $W \rightarrow f_1\bar{f}_2$ ;

The system was already used for calculation of EWRC to  $e^+e^- \rightarrow t\bar{t}$  (as described in Section 3).

In case if the project will be approved by scientific community (I mean, it will be decided that it is worth doing, since there is certain competition with FeynArts, see, for instance [5]), during next two years (2001–2002) it would be feasible to realize next steps:

1. extension of computation of one-loop scalar form factors for all SM  $1 \rightarrow 2$  decays and  $2f \rightarrow 2f$  processes of experimental interest (**REI criterion** – *Reactions of Experimental Interest*. By this we mean that in the initial state could be only  $e^\pm, \gamma$ , muons, neutrinos and partons).
2. extension of availability of generation and reduction of FD for all  $2 \rightarrow 2$  processes;
3. realization of the step: from form factors to helicity amplitudes;
4. realization of the step: from amplitudes to realistic observables;
5. solution of the problem of automatic generation of codes for numerical calculation of form factors, amplitudes and observables;
6. creation of codes for calculation of one-loop amplitudes for SM REI  $2 \rightarrow 3$  processes;
7. user support of CalcPHEP system; creation of user-friendly environment on the site.

### 3 New calculation for $e^+e^- \rightarrow t\bar{t}$

The process  $e^+e^- \rightarrow t\bar{t}$  is studied already about ten years in connection with experiments at future linear colliders, see for instance recent review [6].

Actually, it is a six-fermion process, however the cross-section  $\sigma(e^+e^- \rightarrow t\bar{t})$  with tops on-mass-shell is an ingredient in various approaches like DPA [7], or the so-called Modified Perturbation Theory (MPT) [8].

Recently we completed a new calculation of the electroweak part of the amplitude of  $e^+e^- \rightarrow t\bar{t}$  process [9] in two gauges,  $R_\xi$  and unitary ones.

There were many studies of  $e^+e^- \rightarrow t\bar{t}$  process in  $\xi = 1$  gauge, see for example [10] and [11].

The **purposes** of this new study are: 1) to explicitly control gauge invariance in  $R_\xi$  and search for gauge invariant subsets of diagrams in fully massive case; 2) to compare with the result in the unitary gauge as an internal cross-check; 3) to propose a way of realization of the step from FD to renormalized amplitudes within CalCPHEP project; 4) to compare with existing in the literature results; 5) to create a **FORT**RAN code for the IBA (Improved Born Approximation) cross-section  $d\sigma/dt \sim |A|^2 = |A_{BORN} + A_{WEAK}|^2$  for subsequent use within MPT and within “algebraic” approach, see Section 4.

### 3.1 Amplitudes in $L, Q, D$ basis

In presence of massive fermions it is convenient to introduce the so-called  $L, Q, D$  basis in which the amplitude may be parameterized with **six** scalar form factors:

$$\begin{aligned}
A_\gamma &= i \frac{4\pi Q_e Q_f}{s} \alpha(s) \gamma_\mu \otimes \gamma_\mu, \\
A_Z &= i \frac{g^2}{16\pi^2} e^2 4I_e^{(3)} I_t^{(3)} \frac{\chi_Z(s)}{s} \left\{ \gamma_\mu \gamma_+ \otimes \gamma_\mu \gamma_+ F_{LL}(s, t) \right. \\
&\quad - 4|Q_e| s_W^2 \gamma_\mu \otimes \gamma_\mu \gamma_+ F_{QL}(s, t) - 4|Q_t| s_W^2 \gamma_\mu \gamma_+ \otimes \gamma_\mu F_{LQ}(s, t) \\
&\quad + 16|Q_e Q_t| s_W^4 \gamma_\mu \otimes \gamma_\mu F_{QQ}(s, t) \\
&\quad \left. - \gamma_\mu \gamma_+ \otimes im_t D_\mu F_{LD}(s, t) + 4|Q_e| s_W^2 \gamma_\mu \otimes im_t D_\mu F_{QD}(s, t) \right\}. \quad (3)
\end{aligned}$$

where  $\gamma_+ = 1 + \gamma_5$ ,  $D_\mu = (p_{\bar{t}} - p_t)_\mu$  and  $\chi_Z(s)$  is the  $\gamma/Z$  propagator ratio. Every form factor in  $R_\xi$  gauge could be represented as a sum of two terms:  $F_{L,Q,D}^\xi(s) = F_{L,Q,D}^{(1)}(s) + F_{L,Q,D}^{add}(s, \xi)$ . First term corresponds to  $\xi = 1$  gauge and the second contains all  $\xi$  dependences and vanishes for  $\xi = 1$ . We checked the cancellation of  $\xi$ 's separately for six subsets of diagrams: 1) with virtual  $\gamma$ 's, i.e. QED subset; 2) —  $Z, \phi^0$ , the so-called  $Z$  cluster, i.e. vertices with wave function renormalization factors, see Fig. 1; 3) —  $H, \phi^0$ , i.e.  $H$  cluster; 4) —  $W, \phi^\pm$ , i.e.  $W$  cluster plus all self-energies and  $WW$  box; 5) four  $Z\gamma$  box diagrams; 6) two  $ZZ$  box diagrams.

### 3.2 Scalar form factors of $Z$ cluster

As an example we discuss scalar form factors of  $Z$  cluster, Fig. 1.

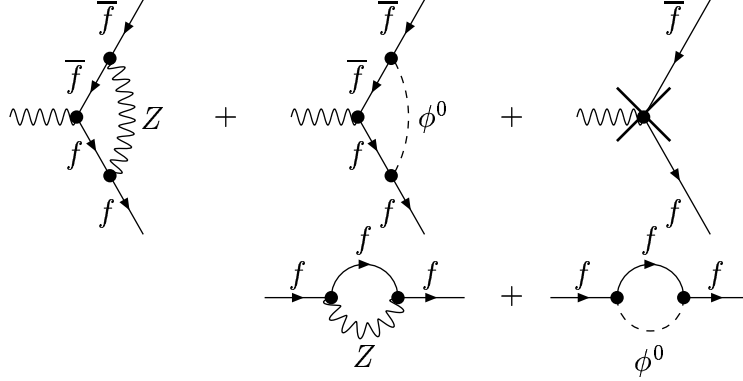


Figure 1:  $Z$  cluster.

They all are gauge invariant objects and four of them are UV-finite. We show pole contributions ( $1/\bar{\epsilon}$ ) explicitly and denote finite quantities with  $\mathcal{F}(s)$ :

$$\begin{aligned}
 F_L^{\gamma Z}(s) &= \mathcal{F}_L^{\gamma Z}(s), \quad F_Q^{\gamma Z}(s) = \mathcal{F}_Q^{\gamma Z}(s), \quad F_D^{\gamma Z}(s) = \mathcal{F}_D^{\gamma Z}(s), \quad F_D^{ZZ}(s) = \mathcal{F}_D^{ZZ}(s), \\
 F_L^{ZZ}(s) &= -\frac{1}{4}r_{tW}\frac{1}{\bar{\epsilon}} + \mathcal{F}_L^{ZZ}(s), \quad F_Q^{ZZ}(s) = -\frac{1}{16}\frac{1}{|Q_t|s_W^2}r_{tW}\frac{1}{\bar{\epsilon}} + \mathcal{F}_Q^{ZZ}(s). \quad (4)
 \end{aligned}$$

We present explicitly only one ‘‘calligraphic’’ quantity:

$$\begin{aligned}
 \mathcal{F}_L^{\gamma Z}(s) &= \frac{Q_t v_t}{c_W^2} \left\{ 2 \left( 2 + \frac{1}{R_Z} \right) M_Z^2 C_0(-m_t^2, -m_t^2, -s; m_t, M_Z, m_t) - L_\mu(m_t^2) \right. \\
 &\quad \left. - 3B_0^F(-s; m_t, m_t) - 2(1 + 4r_{tz}) \frac{M_Z^2}{4m_t^2 - s} L_{ab}(0, m_t, M_Z) \right. \\
 &\quad \left. + 2B_0^F(-m_t^2; m_t, M_Z) + \frac{1}{r_{tz}} \left[ B_0^F(-m_t^2; m_t, M_Z) + L_\mu(M_Z^2) - 1 \right] \right\}, \quad (5)
 \end{aligned}$$

where we introduced definitions:

$$\begin{aligned}
 s_W^2 &= 1 - c_W^2, \quad c_W^2 = \frac{M_W^2}{M_Z^2}, \quad R_Z = \frac{M_Z^2}{s}, \quad r_{tz} = \frac{m_t^2}{M_Z^2}, \quad L_\mu(M^2) = \ln \frac{M^2}{\mu^2}, \\
 L_{ab}(M_1, M_2, M_3) &= \left( 1 + \frac{M_1^2}{M_3^2} \right) M_3^2 C_0(-m_t^2, -m_t^2, -s; M_3, M_2, M_3) \\
 &\quad - B_0^F(-s; M_3, M_3) + B_0^F(-m_t^2; M_2, M_3), \quad (6)
 \end{aligned}$$

and

$$B_0(-s; M_1, M_2) = \frac{1}{\varepsilon} + B_0^F(-s; M_1, M_2). \quad (7)$$

We emphasize that we leave t'Hooft scale parameter  $\mu$  in our formulae unfixed, retaining an opportunity to control  $\mu$ -independence (and therefore UV-finiteness) in numerical realization of one-loop form factors, providing thereby an additional cross-check.

In [9] we present many examples of numerics which exhibit very good level of agreement between ZFITTER and new code.

## 4 “Algebraic” approach to multi-loops

There is a lot of algebraic structure in Feynman diagrams, and the idea is to exploit it to the maximum. That this is indeed possible was discovered in the so-called integration-by-parts, *i-b-p*, algorithm [12]–[13]. It proved to be hugely successful — many well-known NNLO calculations in QCD rely on *i-b-p*. In 1996, F.Tkachov came up with a mathematical result and a scenario to attack arbitrary multi-loop diagrams [14]. It is far from being clear whether it is possible to use it even for simplest 2-loop diagrams. First of all, worth trying it for a familiar one-loop setting. We did it for the scalar form factors  $\mathcal{F}_{L,Q,D}^{\gamma Z}(s)$  of  $Z$  cluster in the process  $e^+e^- \rightarrow t\bar{t}$ , see Fig. 1. Here are some results we have recently obtained [15].

### 4.1 Passarino–Veltman $C_0$ in variables of simplex

We begin with the usual PV function  $C_0$ , see, for instance [3]:

$$i\pi^2 C_0(p_1^2, p_2^2, Q^2; m_1, m, m_2) = \mu^{4-n} \int d^n q \frac{1}{d_0 d_1 d_2}, \quad (8)$$

where

$$\begin{aligned} d_0 &= q^2 + m_1^2 - i\epsilon, & d_1 &= (q + p_1)^2 + m^2 - i\epsilon, & d_2 &= (q + Q)^2 + m_2^2 - i\epsilon, \\ Q &= p_1 + p_2, & Q^2 &= (p_1 + p_2)^2 = -s. \end{aligned} \quad (9)$$

It is very useful to write it down in the so-called *simplex* variables. After standard calculations, we arrive at an integral over two Feynman parameters:

$$C_0(p_1^2, p_2^2, Q^2; m_1, m, m_2) = \int_0^1 dx \int_0^{1-x} dy \frac{1}{P_{x,y}}, \quad (10)$$

with

$$P_{x,y} = -x^2 p_1^2 - y^2 p_2^2 + x y (Q^2 - p_1^2 - p_2^2) + x (p_1^2 + m_1^2 - m^2) + y (p_2^2 + m_2^2 - m^2) + m^2. \quad (11)$$

We emphasize that both the integration domain (*unit triangle*) and integrand are symmetric with respect to interchange of  $x \leftrightarrow y$  (and  $p_1 \leftrightarrow p_2$ ,  $m_1 \leftrightarrow m_2$ ).

Imagine now that the integral of Eq.(10) is in calculable analytically (dilogarithms are not discovered!) and that the only way to compute it is a numerical integration over Feynman parameters. Representations of Eq.(10) is very bad for numerical treatment due to existence of zeroes in polynomial  $P_{x,y}$ .

## 4.2 “Lifting” of polynomial powers

As was proved in [14], there exists a differential operator with an aid of which it is always possible to “lift” the power of a polynomial:

$$P_{x,y}^k = \frac{1}{\Delta} \left[ 1 - \frac{(x + A_x) \partial_x + (y + A_y) \partial_y}{2(k+1)} \right] P_{x,y}^{k+1}. \quad (12)$$

Repeating procedure recursively several times, it is possible to transform any negative power into any positive power. Eq.(12) involves some *new determinant*  $\Delta$ . For instance, for  $P_{x,y}$  one has:

$$\begin{aligned} P_{x,y} \equiv P_{x,y}(m_t, M_z, m_t) &= Q^2 xy + m_t^2 (x+y)^2 + M_z^2 (1-x-y), \\ P_{x,y}(m_t, M_z, m_t) &\rightarrow \Delta = \frac{\Delta(m_t, M_z, m_t)}{\Delta_3}, \quad A_{x(y)} = Q^2 \frac{M_z^2}{4\Delta_3}, \end{aligned} \quad (13)$$

where

$\Delta_3 = -\frac{1}{4}Q^2(Q^2 + 4m_t^2)$  is the Gram determinant for our 3-point function. Vector  $A$  and new determinant  $\Delta$  are inherent to a given diagram, contrary to Gram determinant which depends only on external momenta, they feel all internal masses of a diagram.

## 4.3 New reduction

Exploiting identities Eq.(12) and a reduction in  $n$ -dimensions which is an alternative to PV reduction, we express the scalar form factors of our  $Z$



cluster in terms of untaken integrals over Feynman parameters. The latter appear *inside* and *outside* polynomials (in powers  $k + \varepsilon$ ,  $k = 1, 0$ ). All  $x$  and  $y$ , which appear outside, may be eliminated yet in  $n$ -dimensions. This is a *reduction* because it reduces expressions to a very limited number of functions which might be termed as *new scalars*. Reduction in  $n$ -dimensions heavily exploits *i-b-p* and the symmetry of simplex.

### 4.3.1 New scalars

In framework of this approach we meet an analog of the usual  $C_0$  and  $B_0$  functions:

$$C_0(k, \mu^2; p_1^2, p_2^2, Q^2; m_1, m, m_2) = \int_0^1 dx \int_0^{1-x} dy P_{x,y}^k \ln \frac{P_{x,y}}{\mu^2}, \quad (14)$$

$$B_0(k, \mu^2; Q^2; M_1, M_2) = \int_0^1 dx P_x^k \ln \frac{P_x}{\mu^2}, \quad (15)$$

where in the most general case:  $P_x = Q^2 x(1-x) + M_1^2 x + M_2^2(1-x)$ .

All new scalars, not only  $B_0$ , depend on the t'Hooft scale  $\mu$ . We use short hand notation for new scalars:

$$\begin{aligned} \hat{L}_k(m_t, M_Z, m_t) &= M_Z^{-2k} C_0(k, \mu^2; -m_t^2, -m_t^2, -s; m_t, M_Z, m_t), \\ \hat{L}_k(m_t, 0, m_t) &= M_Z^{-2k} B_0(k, \mu^2; -s; m_t, m_t), \\ \hat{L}_k(m_t, M_Z, 0) &= M_Z^{-2k} B_0(k, \mu^2; -m_t^2; m_t, M_Z), \end{aligned} \quad (16)$$

Next, we present some results which were derived after application of the procedure *one* and *two times*.

### 4.3.2 “Once lifted” expressions

We show only one scalar form factor  $F_{L,1}^{\gamma Z}$  as a typical example:

$$\begin{aligned} F_{L,1}^{\gamma Z} &= \frac{Q_t v_t}{c_w^2} \left\{ 2\hat{L}_0(m_t, M_Z, m_t) - L_\mu(m_t) + 1 - \frac{1}{r_{tz}} \left[ \hat{L}_0(m_t, M_Z, 0) \right. \right. \\ &\quad \left. \left. - L_\mu(M_Z) + 1 \right] - \frac{1}{R_Z} \left( \hat{L}_0(m_t, 0, m_t) - 2\hat{L}_0(m_t, M_Z, m_t) - 1 \right) \right\}. \end{aligned} \quad (17)$$

Expression for  $F_{L,1}^{\gamma Z}$  is remarkably compact, cf. Eq.(5), and does not contain any determinants at all (the latter property is not typical, however).

“**Twice lifted**” expressions have a similar structure, however, expressions blow up with increasing the number of recursions.

In [15] we show results of numerical computation of  $F_{L,S}^{\gamma Z}$ , i.e. standard approach giving an analytic expressions Eq.(5) in terms of dilogs; and of numerical computation of  $F_{L,1}^{\gamma Z}$ , i.e. “once lifted” expressions Eq.(17) and  $F_{L,2}^{\gamma Z}$ , i.e. “twice lifted”. We also show numbers for  $F_{Q,(S,1,2)}^{\gamma Z}$ . Numbers show 9-10 digit agreement for Re part and 5 digits — for Im part at  $k = 1$ . This demonstrates that approach succeeds at one loop.

## 5 Outlook

In my opinion, nowadays in the field of theoretical support of experiments, it is reasonable to work in two directions:

1) Complete one-loop SM corrections for processes with number of particles in the final state  $N_f \geq 3$  (for applications at TEVATRON, LHC, LC,  $\mu$ -factory). Our project CalcPHEP belongs to this direction. In connection with this, one should say that if one wants to attack a problem of a certain level complexity, not starting just from the previous level, all the intermediate levels have to be worked through. This will considerably delay the realization of the project. Furthermore, complete one-loop corrections for the process  $e^+e^- \rightarrow f\bar{f}\gamma$  (where  $f$  is any fermion, including electron) is a part of the two-loop program for the  $Z$  resonance, so we naturally come to the second direction, namely:

2) Work towards two-loop precision level control of HEP observables (for applications at GigaZ LC option). “Algebraic” approach for two-loops belongs to this direction. It undoubtedly works and possesses appealing features at one loop. Beyond one loop, there are huge algebraic difficulties; explicit solution has never been found. F.Tkachov currently explores scenarios that look bizarre enough to hold promises for such a complicated problem. Are there any chances of success in a foreseeable future? One may count only on: 1) a non-standard F.Tkachov’s expertise which integrates a non-trivial understanding of the math involved with a considerable skill in algorithm design and software engineering; and on 2) our community intentions to continue digging in this direction, in particular, now in the two-loop field (see Giampiero Passarino’s talk at this Symposium).

*Happy birthday, Alberto!*

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