Radiative Corrections to Neutrino Deep Inelastic Scattering Revisited

A.B. Arbuzov [a], D.Yu. Bardin [b], and L.V. Kalinovskaya [b]

[a] *Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141980 Russia*

[b] *Dzhelepov Laboratory of Nuclear Problems, JINR, Dubna, 141980 Russia*

Abstract

Radiative corrections to neutrino deep inelastic scattering are revisited. One–loop electroweak corrections are re–calculated within the automatic SANC system. Terms with mass singularities are treated including higher order leading logarithmic corrections. Scheme dependence of corrections due to weak interactions is investigated. The results are implemented into the data analysis of the NOMAD experiment. The present theoretical accuracy in description of the process is discussed.

Key words: neutrino, deep inelastic scattering, radiative corrections
PACS: 13.15.+g Neutrino interactions, 12.15.Lk Electroweak radiative corrections, 13.40.Ks Electromagnetic corrections to strong- and weak-interaction processes

1 Introduction

Modern experiments such as NOMAD [1], NuTeV [2], and CHORUS [3] made a serious step forward in studies of neutrino deep inelastic scattering. Their precision measurements made it necessary to update the accuracy level of the theoretical description of the process. Important ingredients for an advanced precision in the theoretical predictions is the calculation of the relevant radiative corrections (RC).

Our study is motivated by the request from the NOMAD experiment. Electroweak (EW) radiative corrections to neutrino–nucleon scattering should have been implemented into the general Monte Carlo system for the experimental data analysis. Certain experimental conditions of particle registration and event selection should have been taken into account. In order to make the relevant subroutine describing the corrections fast, we had to look for an analytical answer (without numerical integrations).

In the present work we reproduce most of the results of unpublished communication [4]. Contrary to the earlier calculation, we used the modern technique of automatic calculations within the SANC system [5,6,7,8] developed for Support of Analytic and Numeric calculations for experiments at Colliders. Besides the one-loop calculation, we consider certain contributions of higher orders and discuss the theoretical uncertainty due to unknown electroweak (EW) corrections for the case of the concrete experimental study.

The paper is organized as follows. In the next section we define the notation and present the Born level distributions. Then we consider different sources of radiative corrections. Special subsections are devoted to quark and muon mass singularities. Numerical results and comparisons are presented in Sect. 4. Possible applications of our formulae are discussed in Conclusions.

2 The Born Cross Section

We will consider the process of deep inelastic scattering (DIS) in the framework of the quark–parton model, assuming that we are in the proper kinematical region ($Q^2 \gg \Lambda^2_{QCD}$).

Here we list the Born level cross sections of neutrino–quark interaction weighted by the quark density functions. For the charge current (CC) scattering processes

$$\nu(k_1) + q_i(p_1) \to l^-(k_2) + q_f(p_2) \quad \text{and} \quad \bar{\nu} + q_i \to l^+ + q_f \tag{1}$$

we have

$$\frac{\mathrm{d}^2\sigma^{\mathrm{Born}}_{\nu CC}}{\mathrm{d}x\mathrm{d}y} = \sigma^0_{CC}, \qquad \frac{\mathrm{d}^2\sigma^{\mathrm{Born}}_{\bar{\nu} CC}}{\mathrm{d}x\mathrm{d}y} = \sigma^0_{CC}(1-y)^2, \tag{2}$$

$$\sigma^0_{CC} = |V_{if}|^2 \frac{G_F^2}{\pi} \hat{s} \frac{M_W^4}{(M_W^2 + \hat{Q}^2)^2} f_i(x, \hat{Q}^2), \tag{3}$$

where M_W is the W-boson mass; $\hat{s}$ is the center–of–mass energy of the neutrino-quark system squared; $|V_{if}|$ is the element of the Cabibbo-Kobayashi-Maskawa quark mixing matrix; $f_i(x, \hat{Q}^2)$ is the density function of the initial quark in the given nucleon. The kinematics [1] is described by the Bjorken variables:

$$y = \frac{\hat{Q}^2}{\hat{s}}, \qquad \hat{Q}^2 = -(p_2 - p_1)^2, \qquad x = \frac{\hat{Q}^2}{yS},$$
$$\hat{s} = (k_1 + p_1)^2 \approx xS, \qquad S = (k_1 + P)^2, \tag{4}$$

[1] We use the $(+,-,-,-)$ metrics, $p = (p^0, \vec{p})$.

where P is the initial nucleon momentum. We will use $m_{1,2,l}$ ($Q_{1,2,l}$) for the masses (charges) of the initial quark, the final quark, and the muon.

For the neutral current (NC) scattering processes

$$\nu(k_1) + q_i(p_1) \to \nu(k_2) + q_f(p_2) \quad \text{and} \quad \bar{\nu} + q_i \to \bar{\nu} + q_f \tag{5}$$

we have

$$\frac{\mathrm{d}^2\sigma^{\text{Born}}_{\nu NC}}{\mathrm{d}x\mathrm{d}y} = \sigma^0_{NC}\left(g_L^2 + g_R^2(1-y)^2 - \frac{2m_1^2 y}{\hat{s}} g_L g_R\right), \tag{6}$$

$$\frac{\mathrm{d}^2\sigma^{\text{Born}}_{\bar{\nu} NC}}{\mathrm{d}x\mathrm{d}y} = \sigma^0_{NC}\left(g_R^2 + g_L^2(1-y)^2 - \frac{2m_1^2 y}{\hat{s}} g_L g_R\right), \tag{7}$$

$$\sigma^0_{NC} = \frac{G_F^2}{\pi}\hat{s}\frac{M_Z^4}{(M_Z^2 + \hat{Q}^2)^2} f_i(x, \hat{Q}^2), \tag{8}$$

where

$$g_L = -\frac{1}{2} + |Q_i|\sin^2\theta_W, \qquad g_R = |Q_i|\sin^2\theta_W, \tag{9}$$

M_Z is the Z-boson mass; and θ_W is the weak mixing angle.

3 Radiative Corrections

The radiatively corrected neutrino DIS cross section can be represented as the sum of the Born distribution with the contributions due to virtual loop diagrams (Virt), soft photon emission (Soft), and hard photon emission (Hard):

$$\frac{\mathrm{d}^2\sigma_i^{\text{Corr.}}}{\mathrm{d}x\ \mathrm{d}y} = \frac{\mathrm{d}^2\sigma_i^{\text{Born}}}{\mathrm{d}x\ \mathrm{d}y}\left(1 + \delta_i^{\text{Virt}} + \delta_i^{\text{Soft}} + \delta_i^{\text{Hard}}\right), \tag{10}$$

where the index i denotes the type of the process under consideration ($\nu q\ CC$, $\bar{\nu} q\ NC$ and so on). This formula assumes calculation of the order $\mathcal{O}(\alpha)$ RC. Certain higher order corrections will be added below as well. We assume also that the momentum transfer square is small compared with the W-boson mass: corrections of the order $\alpha Q^2/M_W^2$ are omitted from the calculation.

3.1 Virtual Corrections

Contributions due to virtual (one–loop) EW corrections are re–calculated by means of the SANC system. All the contributing one–loop diagrams were calculated in the frame of the Standard Model. Omitting the terms suppressed by the Q^2/M_W^2 ratio allowed us to get the short analytical answer.

The virtual correction to the CC neutrino–quark scattering reads

$$
\begin{aligned}
\delta^{\rm Virt}_{\nu CC} = \frac{\alpha}{\pi}\Biggl\{ & -\Biggl[\frac{1}{2}Q_1^2\ln\frac{\hat{Q}^2}{m_1^2} + \frac{1}{2}(1+Q_1)^2\ln\frac{\hat{s}}{m_2^2} + \frac{1}{2}\ln\frac{\hat{s}}{m_l^2} \\
& + \left(Q_1 + \frac{Q_1^2}{2}\right)\ln y - Q_1\ln(1-y) - 1 - Q_1 - Q_1^2\Biggr]\ln\frac{\hat{s}}{\lambda^2} \\
& + \frac{1}{4}\Biggl[Q_1^2\ln^2\frac{\hat{Q}^2}{m_1^2} + (1+Q_1)^2\ln^2\frac{\hat{s}}{m_2^2} + \ln^2\frac{\hat{s}}{m_l^2} + \ln\frac{\hat{s}}{m_l^2} \\
& + Q_1^2(1-2\ln y)\ln\frac{\hat{Q}^2}{m_1^2} + (1+Q_1)^2\ln\frac{\hat{s}}{m_2^2}\Biggr] \\
& - \frac{1}{4}\Biggl[Q_1(2+Q_1)\ln^2 y - 2Q_1\ln^2(1-y) - Q_1(6+5Q_1)\ln y \\
& + 6Q_1\ln(1-y) - 4(2+Q_1)^2\zeta_2 + 1 + Q_1 + 8Q_1^2\Biggr] \\
& - \frac{3}{2}(1+Q_1)\ln\frac{\hat{s}}{M_Z^2}\Biggr\}, \qquad (11) \\
& \zeta_2 = \mathrm{Li}_2(1) = \frac{\pi^2}{6}, \qquad \mathrm{Li}_2(x) = -\int\limits_0^1 \frac{\ln(1-xy)}{y}\mathrm{d}y,
\end{aligned}
$$

where λ is the auxiliary photon mass, $\lambda \ll m_{1,2,l}$. Note that in the above expression we used the explicit value $Q_l = -1$ and eliminated the final state quark charge by applying the charge conservation law, $Q_2 = Q_1 - Q_l$.

For the CC antineutrino–quark scattering we get

$$
\begin{aligned}
\delta^{\rm Virt}_{\bar{\nu} CC} = \frac{\alpha}{\pi}\Biggl\{ & -\Biggl[\frac{1}{2}Q_1^2\ln\frac{\hat{Q}^2}{m_1^2} + \frac{1}{2}(1-Q_1)^2\ln\frac{\hat{s}}{m_2^2} + \frac{1}{2}\ln\frac{\hat{s}}{m_l^2} \\
& - \left(Q_1 - \frac{Q_1^2}{2}\right)\ln y + Q_1\ln(1-y) - 1 + Q_1 - Q_1^2\Biggr]\ln\frac{\hat{s}}{\lambda^2} \\
& + \frac{1}{4}\Biggl[Q_1^2\ln^2\frac{\hat{Q}^2}{m_1^2} + (1-Q_1)^2\ln^2\frac{\hat{s}}{m_2^2} + \ln^2\frac{\hat{s}}{m_l^2} + \ln\frac{\hat{s}}{m_l^2}
\end{aligned}
$$

$$
+Q_1^2(1-2\ln y)\ln\frac{\hat{Q}^2}{m_1^2}+(1-Q_1)^2\ln\frac{\hat{s}}{m_2^2}\Big]
$$

$$
+\frac{1}{4}\Big[Q_1(2-Q_1)\ln^2 y-2Q_1\ln^2(1-y)-Q_1(6-5Q_1)\ln y
$$

$$
+4(4-Q_1+Q_1^2)\zeta_2-8+15Q_1-8Q_1^2\Big]-\frac{3}{2}Q_1\ln\frac{\hat{s}}{M_Z^2}\Big\}. \qquad (12)
$$

For the pure QED part of the virtual corrections to the NC neutrino (and antineutrino) scattering we have

$$
\delta_{NC}^{\mathrm{Virt}}=\frac{\alpha}{\pi}Q_1^2\Big\{-\frac{1}{2}\ln\frac{\hat{Q}^2}{\lambda^2}\Big(\ln\frac{\hat{Q}^2}{m_1^2}+\ln\frac{\hat{Q}^2}{m_2^2}-2\Big)+\frac{1}{4}\ln^2\frac{\hat{Q}^2}{m_1^2}+\frac{1}{4}\ln^2\frac{\hat{Q}^2}{m_2^2}
$$

$$
+\frac{1}{4}\Big(\ln\frac{\hat{Q}^2}{m_1^2}+\ln\frac{\hat{Q}^2}{m_2^2}\Big)-2+\zeta_2\Big\}. \qquad (13)
$$

The weak part of the virtual correction to the NC case is included in the definition of the effective electroweak couplings:

$$
g_{L(R)}\to\tilde{g}_{L(R)},\quad \tilde{g}_L=\rho\Big(-\frac{1}{2}+|Q_i|\kappa\sin^2\theta_W\Big),\quad \tilde{g}_R=\rho\kappa|Q_i|\sin^2\theta_W, \qquad (14)
$$

where ρ and κ are the electroweak form factors,

$$
\rho=\frac{3}{4}\Big\{\frac{1}{s_W^2}\ln\left(\frac{M_W^2}{M_Z^2}\right)+\frac{M_H^2}{M_W^2}\left[\frac{1}{(1-M_H^2/M_Z^2)}\ln\left(\frac{M_H^2}{M_Z^2}\right)-\frac{1}{(1-M_H^2/M_W^2)}\ln\left(\frac{M_H^2}{M_W^2}\right)\right]
$$

$$
+1-4I_f^{(3)}-\frac{4}{c_W^2}\left(\frac{1}{2}I_f^{(3)}-s_W^2c_f+4I_f^{(3)}s_W^4c_f^2\right)+\frac{m_t^2}{M_W^2}\Big\}, \qquad (15)
$$

$$
\kappa=-\frac{c_W^2}{s_W^2}\Delta\rho-\frac{7}{2}+3I_f^{(3)}-\frac{2}{3}c_W^2+\frac{3}{c_W^2}\left(\frac{1}{2}I_f^{(3)}-\frac{3}{2}s_W^2c_f+4I_f^{(3)}s_W^4c_f^2\right)
$$

$$
+2B_f(Q^2;m_f,m_f)-\Pi_{Z\gamma}^{\mathrm{fer}}(Q^2), \qquad (16)
$$

where $I_f^{(3)}$, c_f, Q_f, v_f and m_f are weak isospin, color factor (1 for leptons, 3 for quarks), charge, vector coupling and mass of a fermion; notation for W,Z,H,t quark masses are evident; s_W and c_W are sine and cosine of weak mixing angle and $s_W^2=1-M_W^2/M_Z^2$; $\Delta\rho$ is Veltman parameter,

$$
\Delta\rho=\frac{1}{M_W^2}\left[\Sigma(M_W^2)-\Sigma(M_Z^2)\right]; \qquad (17)
$$

function $B_f(Q^2;m_f,m_f)$ is a useful combination of the finite parts of the standard

Passarino–Veltman functions (without UV pole $1/\bar{\epsilon}$)

$$B_f(Q^2; m_f, m_f) = 2\left[B_{21}(Q^2; m_f, m_f) + B_1(Q^2; m_f, m_f)\right]; \qquad (18)$$

and finally $\Pi^{\rm fer}_{Z\gamma}(Q^2)$ is the γZ mixing operator

$$\Pi^{\rm fer}_{Z\gamma}(Q^2) = 2\sum_f c_f\, Q_f\, v_f\, B_f(Q^2; m_f, m_f). \qquad (19)$$

Note, that the three last quantities are taken at $\mu = M_W$, with μ being the t'Hooft scale. More about the calculation and renormalization scheme applied here can be found in the book [9].

3.2 Soft Photon Radiation

Emission of a soft photon in neutrino DIS can be described in the standard way by the accompanying radiation factors:

$$\delta^{\rm Soft}_{NC} = -Q_1^2 \frac{\alpha}{4\pi^2}\int \frac{{\rm d}^3 k}{\omega}\left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k}\right)^2, \qquad (20)$$

$$\delta^{\rm Soft}_{CC} = -\frac{\alpha}{4\pi^2}\int \frac{{\rm d}^3 k}{\omega}\left(Q_1\frac{p_1}{p_1 k} - Q_2\frac{p_2}{p_2 k} - Q_l\frac{k_2}{k_2 k}\right)^2. \qquad (21)$$

We consider the problem in the rest reference frame of the final quark, $\vec{p}_2 = 0$, which is equivalent (in the soft photon limit) to the so–called R-reference frame, used below in the evaluation of the hard photon contribution. The soft photon energy, ω, is limited by the parameter $\bar{\omega}$, which is assumed to be small compared to the large energy scale: $\bar{\omega} \ll \sqrt{Q^2}$. List of the relevant integrals is given in Appendix A. For the charged current we get

$$\begin{aligned}
\delta^{\rm Soft}_{\nu CC} = \frac{\alpha}{\pi}\Bigg\{ \Bigg[&\frac{1}{2}Q_1^2 \ln\frac{\hat{Q}^2}{m_1^2} + \frac{1}{2}(1+Q_1)^2 \ln\frac{\hat{s}}{m_2^2} + \frac{1}{2}\ln\frac{\hat{s}}{m_l^2} \\
&+ Q_1\left(1 + \frac{1}{2}Q_1\right)\ln y - Q_1 \ln(1-y) - 1 - Q_1 - Q_1^2 \Bigg] \ln\frac{\hat{s}}{\lambda^2} \\
&- \Bigg[Q_1^2 \ln\frac{\hat{Q}^2}{m_1^2} + (1+Q_1)^2 \ln\frac{\hat{s}}{m_2^2} + \ln\frac{\hat{s}}{m_l^2} \\
&+ Q_1(2+Q_1)\ln y - 2Q_1\ln(1-y) - 2(1+Q_1+Q_1^2) \Bigg] \ln\frac{\hat{s}}{2\bar{\omega} m_2}
\end{aligned}$$

$$
-\frac{1}{4}\Bigl[Q_1^2 \ln^2\frac{\hat{Q}^2}{m_1^2} - (1+Q_1)^2\ln^2\frac{\hat{s}}{m_2^2} + \ln^2\frac{\hat{s}}{m_l^2} - 2Q_1^2(1-\ln y)\ln\frac{\hat{Q}^2}{m_1^2}
$$

$$
+2(1+Q_1)^2\ln\frac{\hat{s}}{m_2^2} - 2\ln\frac{\hat{s}}{m_l^2}\Bigr] - \frac{1}{4}Q_1(2+Q_1)\ln^2 y - \frac{1}{2}Q_1\ln^2(1-y)
$$

$$
+Q_1\ln y\ln(1-y) + \frac{1}{2}Q_1^2\ln y - (1+Q_1+Q_1^2)\zeta_2 + (1+Q_1)^2\Bigr\}, \tag{22}
$$

$$
\delta^{\mathrm{Soft}}_{\bar{\nu}CC} = \frac{\alpha}{\pi}\Bigl\{\Bigl[\frac{1}{2}Q_1^2\ln\frac{\hat{Q}^2}{m_1^2} + \frac{1}{2}(1-Q_1)^2\ln\frac{\hat{s}}{m_2^2} + \frac{1}{2}\ln\frac{\hat{s}}{m_l^2}
$$

$$
-Q_1\left(1-\frac{1}{2}Q_1\right)\ln y + Q_1\ln(1-y) - 1 + Q_1 - Q_1^2\Bigr]\ln\frac{\hat{s}}{\lambda^2}
$$

$$
-\Bigl[Q_1^2\ln\frac{\hat{Q}^2}{m_1^2} + (1-Q_1)^2\ln\frac{\hat{s}}{m_2^2} + \ln\frac{\hat{s}}{m_l^2}
$$

$$
-Q_1(2-Q_1)\ln y + 2Q_1\ln(1-y) - 2(1-Q_1+Q_1^2)\Bigr]\ln\frac{\hat{s}}{2\bar{\omega}m_2}
$$

$$
-\frac{1}{4}\Bigl[Q_1^2\ln^2\frac{\hat{Q}^2}{m_1^2} - (1-Q_1)^2\ln^2\frac{\hat{s}}{m_2^2} + \ln^2\frac{\hat{s}}{m_l^2} - 2Q_1^2(1-\ln y)\ln\frac{\hat{Q}^2}{m_1^2}
$$

$$
+2(1-Q_1)^2\ln\frac{\hat{s}}{m_2^2} - 2\ln\frac{\hat{s}}{m_l^2}\Bigr] + \frac{1}{4}Q_1(2-Q_1)\ln^2 y + \frac{1}{2}Q_1\ln^2(1-y)
$$

$$
-Q_1\ln y\ln(1-y) + \frac{1}{2}Q_1^2\ln y - (1-Q_1+Q_1^2)\zeta_2 + (1-Q_1)^2\Bigr\}. \tag{23}
$$

Soft corrections for the NC case are given by

$$
\delta^{\mathrm{Soft}}_{NC} = -\frac{\alpha}{\pi}Q_1^2\Bigl\{\ln\frac{2\bar{\omega}}{\lambda}\Bigl(2-\ln\frac{\hat{Q}^2}{m_1^2}-\ln\frac{\hat{Q}^2}{m_2^2}\Bigr)
$$

$$
+\frac{1}{4}\Bigl(\ln\frac{\hat{Q}^2}{m_1^2}+\ln\frac{\hat{Q}^2}{m_2^2}\Bigr)^2 - \frac{1}{2}\Bigl(\ln\frac{\hat{Q}^2}{m_1^2}+\ln\frac{\hat{Q}^2}{m_2^2}\Bigr) + \zeta_2 - 1\Bigr\}. \tag{24}
$$

The above formula is valid both for the neutrino–quark and antineutrino–quark NC scattering processes.

3.3 Hard Photon Radiation

In the case of hard photon emission, like in the process

$$
\nu(k_1) \ + \ q_i(p_1) \ \to \ l^-(k_2) \ + \ q_f(p_2) \ + \ \gamma(k), \tag{25}
$$

we have to extend the list of kinematical variables:

$$\tilde{p}_2 = p_2 + k, \qquad \tilde{Q}^2 = -2p_1(p_2+k), \qquad \tilde{M}^2 = -(p_2+k)^2. \tag{26}$$

Note, that in the case without real photon radiation $\tilde{Q}^2 = \hat{Q}^2$. Therefore in what follows we can use $\tilde{Q}^2$ instead of $\hat{Q}^2$. The calculations of the hard photon contribution were evaluated also in the environment of the SANC system.

The angular phase space of the hard photon is $\mathrm{d}\Omega_R = \mathrm{d}\cos\theta_R \mathrm{d}\varphi_R$ in the R-system of reference, where $\vec{k} + \vec{p}_2 = 0$.

For the charged current νq scattering we have

$$\begin{aligned}
\delta^{\mathrm{Hard}}_{\nu CC} = \frac{\alpha}{\pi}\bigg\{ & \bigg[Q_1^2 \ln\frac{\tilde{Q}^2}{m_1^2} + (1+Q_1)^2 \ln\frac{\hat{s}}{m_2^2} + \ln\frac{\hat{s}}{m_l^2} \\
& + Q_1(2+Q_1)\ln y - 2Q_1\ln(1-y) - 2(1+Q_1+Q_1^2)\bigg] \ln\frac{\hat{s}}{2\bar{\omega}m_2} \\
& - \frac{1}{2}(1+Q_1)^2\ln^2\frac{\hat{s}}{m_2^2} - Q_1^2\left(\frac{17}{12} - \ln y\right)\ln\frac{\tilde{Q}^2}{m_1^2} \\
& + \frac{1}{4}(1+Q_1)^2\ln\frac{\hat{s}}{m_2^2} + \frac{1}{2}\Big[2\ln y - \ln(1-y) - y\Big]\ln\frac{\hat{s}}{m_l^2} \\
& - \left(\frac{1}{2} - Q_1 - \frac{1}{2}Q_1^2\right)\ln^2 y - \frac{1}{2}\ln^2(1-y) + \left(\frac{1}{2} - 2Q_1\right)\ln(1-y)\ln y \\
& - \left(\frac{7}{4} - \frac{1}{2}y + \frac{3}{2}Q_1 + \frac{7}{4}Q_1^2\right)\ln y \\
& + \left(1 - y + \frac{3}{2}Q_1\right)\ln(1-y) - \left(\frac{3}{2} + 2Q_1\right)\mathrm{Li}_2\,(y) - (1+Q_1)^2\zeta_2 \\
& - \frac{1}{4}\bigg[1 - 5y - 5Q_1 - \left(\frac{95}{18} - \frac{5}{3}y + \frac{1}{6}y^2\right)Q_1^2\bigg]\bigg\}.
\end{aligned} \tag{27}$$

Hard photon contribution to the $\bar{\nu}q$ CC scattering reads

$$\begin{aligned}
\delta^{\mathrm{Hard}}_{\bar{\nu} CC} = \frac{1}{(1-y)^2}\frac{\alpha}{\pi}\bigg\{ & (1-y)^2\bigg[Q_1^2 \ln\frac{\tilde{Q}^2}{m_1^2} + (1-Q_1)^2 \ln\frac{\hat{s}}{m_2^2} + \ln\frac{\hat{s}}{m_l^2} \\
& + Q_1(Q_1-2)\ln y + 2Q_1\ln(1-y) - 2(1-Q_1+Q_1^2)\bigg] \ln\frac{\hat{s}}{2\bar{\omega}m_2} \\
& - \frac{1}{2}(1-Q_1)^2(1-y)^2\ln^2\frac{\hat{s}}{m_2^2} + \frac{1}{4}(1-Q_1)^2(1-y)^2\ln\frac{\hat{s}}{m_2^2} \\
& - Q_1^2(1-y)^2\left(\frac{17}{12} - \ln y\right)\ln\frac{\tilde{Q}^2}{m_1^2} + \Big[(1-y)^2\ln\frac{y}{1-y} + y - \frac{3}{4}y^2\Big]\ln\frac{\hat{s}}{m_l^2} \\
& - \frac{1}{2}(1-y)^2\left(1 + 2Q_1 - Q_1^2\right)\ln^2 y - (1-y)^2\ln^2(1-y) \\
& + (1-y)^2\left(1+2Q_1\right)\ln(1-y)\ln y + \bigg(-\frac{7}{4} + \frac{5}{2}y + \frac{3}{2}(1-y)^2 Q_1 - y^2
\end{aligned}$$

$$
-\frac{7}{4}(1-y)^2Q_1^2\Big)\ln y + \Big(\frac{7}{4}-\frac{5}{2}y+y^2-\frac{3}{2}(1-y)^2Q_1\Big)\ln(1-y)
$$
$$
-(1-y)^2\,(1-2Q_1)\,\mathrm{Li}_2\,(y)-(1-y)^2(1-Q_1)^2\zeta_2-\frac{1}{4}+yQ_1
$$
$$
-\frac{20}{9}yQ_1^2-y+\frac{17}{18}y^2Q_1^2+y^2-\frac{5}{4}Q_1+\frac{95}{72}Q_1^2\Big\}. \qquad (28)
$$

For the neutral current scattering we have

$$
\delta_{\nu NC}^{\mathrm{Hard}} = \frac{\alpha}{\pi}Q_1^2\Big\{\ln\frac{\hat{s}}{2\bar{\omega}m_2}\Big(\ln\frac{\hat{s}}{m_2^2}+\ln\frac{\tilde{Q}^2}{m_1^2}+\ln y-2\Big)+\ln y\ln\frac{\tilde{Q}^2}{m_1^2}
$$
$$
-\frac{17}{12}\ln\frac{\tilde{Q}^2}{m_1^2}-\frac{1}{2}\ln^2\frac{\hat{s}}{m_2^2}+\frac{1}{4}\ln\frac{\hat{s}}{m_2^2}+\frac{1}{2}\ln^2 y-\zeta_2-\frac{7}{4}\ln y
$$
$$
+\frac{17}{18}+\frac{1}{g_L^2+g_R^2(1-y)^2}\Big(\frac{1}{3}(1-y)(g_L^2+g_R^2)+\frac{1}{24}(g_L^2(1-y)^2+g_R^2)\Big)\Big\}. \qquad (29)
$$

The corresponding correction to the NC antineutrino–quark scattering, $\delta_{\bar{\nu}NC}^{\mathrm{Hard}}$, can be obtained from the above equation by the substitution $g_L^2 \leftrightarrow g_R^2$.

3.4 Quark Mass Singularities

In the sum of the soft and hard photonic corrections, the auxiliary parameter $\bar{\omega}$ (soft-hard separator) cancels out. The infrared singular terms (with logarithm of λ) cancel out in the sum of the virtual and soft contributions. Moreover in the total sum, the large logarithms (mass singularities) with the final state quark mass m_2 disappear in accordance with the Kinoshita–Lee-Nauenberg theorem [10,11].

Large logarithms containing the initial quark mass, $\ln(\hat{Q}^2/m_1^2)$, remain in the sum of all contributions. But these logs have been already effectively taken into account in the parton density functions (PDFs). In fact, QED radiative corrections to the quark line are usually not taken into account in procedures of PDF extraction. Moreover, the leading log behaviors of the QED and QCD DGLAP evolution of PDFs in the leading order are proportional to each other. So one gets evolution of PDFs with an effective coupling constant

$$
\alpha_s^{\mathrm{eff}} \approx \alpha_s + \frac{Q_1^2}{C_F}\alpha, \qquad (30)
$$

where α_s is the strong coupling constant, and C_F is the QCD color factor. The nontrivial difference between the QED evolution and the QCD one starts to appear from higher orders, and the corresponding numerical effect is small compared to the remaining QCD uncertainties in PDFs [12,13,14]. See also Ref. [15] for a discussion with respect to the process under consideration.

The best approach to the whole problem would be to re–analyze all the experimental DIS data taking into account QED corrections to the quark line at least at the next–to–leading order. But for the present task we can limit ourselves with application of the $\overline{\rm MS}$ subtraction scheme [16] to the QED part of the radiative corrections for the process under consideration. This leads to a shift of the initial quark mass singularities (with a certain constant term) from our result into the corresponding quark density function. The latter should be also taken in the $\overline{\rm MS}$ scheme [2]. In fact, using the initial condition for the non–singlet NLO QED quark structure function (which coincides with the QCD one with the trivial substitution $C_F\alpha_s \to Q_1^2\alpha$, see Ref. [17]), we get the following expression for the terms to be subtracted from the full calculation with massive quarks:

$$
\begin{aligned}
\delta_{\overline{\rm MS}} &= Q_1^2\frac{\alpha}{2\pi}\int\limits_0^1 {\rm d}x\; x\left[\frac{1+x^2}{1-x}\left(\ln\frac{\hat{Q}^2}{m_1^2} - 1 - 2\ln(1-x)\right)\right]_+ \\
&= Q_1^2\frac{\alpha}{2\pi}\left(-\frac{4}{3}\ln\frac{\hat{Q}^2}{m_1^2} - \frac{17}{9}\right). \qquad (31)
\end{aligned}
$$

3.5 The Muon Mass Singularity

In the case of CC scattering, the large logarithms singular in the limit $m_l \to 0$ remain in the final answer. These terms are in agreement with the prediction of the renormalization group approach. In fact, they can be described by the electron (muon) fragmentation (structure) function approach [18,19,20]:

$$
{\rm d}\sigma^{LL} = \sum_j {\rm d}\tilde{\sigma}_j \otimes D_{\mu j}(\xi, Q^2), \qquad (32)
$$

where ${\rm d}\tilde{\sigma}_j$ is the differential cross section of neutrino DIS with production of particle j ($j = \mu,\ \gamma$, etc.); $D(\xi, Q^2)$ describes the probability to find a muon with the relative energy fraction ξ in particle j; sign $\otimes$ stands for the convolution operation.

For our purposes it is enough to consider only the leading log approximation in the $\mathcal{O}\left(\alpha L\right)$ and $\mathcal{O}\left(\alpha^2 L^2\right)$ orders including the contribution of electron–positron pairs. Under these conditions we can write the QED fragmentation function in the form:

$$
D_{\mu\mu}^{NS}(\xi, Q^2) = \delta(1-\xi) + \frac{\alpha}{2\pi}LP^{(0)}(\xi) + \frac{1}{2}\left(\frac{\alpha}{2\pi}L\right)^2 P^{(0)}(\xi)\otimes P^{(0)}(\xi)
$$

[2] A similar consideration can be performed in the DIS and other schemes as well.

$$+\frac{1}{3}\left(\frac{\alpha}{2\pi}L\right)^2 P^{(0)}(\xi) + \mathcal{O}\left(\alpha, \alpha^2 L, \alpha^3 L^3\right), \tag{33}$$

where L is the so–called large logarithm, and $P^{(0)}(\xi)$ is the lowest order non–singlet splitting function:

$$P^{(0)}(\xi) = \left[\frac{1+\xi^2}{1-\xi}\right]_+ . \tag{34}$$

For the definition of the plus prescription and the convolution operation see for instance Refs. [21,20].

Differential cross section $\mathrm{d}\hat{\sigma}_j$ (for $j=\mu$) is the Born level distributions (2) with a shifted value of variable y:

$$\tilde{y} = \frac{\xi + y - 1}{\xi}, \tag{35}$$

which provides the proper value of variable y after the fragmentation stage under the adopted condition for particle registration. Convolution with the $P^{(0)}$ splitting function gives the following first order leading logarithmic corrections:

$$\delta^{(1)LL}_{\nu CC} = \frac{\alpha}{2\pi}L\biggl(2\ln y - \ln(1-y) + \frac{3}{2} - y\biggr), \tag{36}$$

$$\delta^{(1)LL}_{\bar{\nu} CC} = \frac{1}{(1-y)^2}\frac{\alpha}{2\pi}L\biggl(2(1-y)^2\ln\frac{y}{1-y} + \frac{3}{2} - y\biggr). \tag{37}$$

We will see that the above expressions reproduce the the leading log part of the complete result for the $\mathcal{O}(\alpha)$ correction in Eqs. (41,42).

By convolution with the $\mathcal{O}(\alpha^2)$ term from Eq. (33), we get the second order leading log corrections to the CC neutrino DIS:

$$\begin{aligned}\delta^{(2)LL}_{\nu CC} &= \frac{1}{2}\left(\frac{\alpha}{2\pi}L\right)^2\biggl[4\ln^2 y - 4\ln y\ln(1-y) + \frac{1}{2}\ln^2(1-y) - 4\zeta_2 \\ &\quad +(6-4y)\ln y + (3y-4)\ln(1-y) + \frac{9}{4}\biggr] + \frac{1}{3}\frac{\alpha}{2\pi}L\delta^{(1)LL}_{\nu CC},\end{aligned} \tag{38}$$

$$\begin{aligned}\delta^{(2)LL}_{\bar{\nu} CC} &= \frac{1}{(1-y)^2}\frac{1}{2}\left(\frac{\alpha}{2\pi}L\right)^2\biggl[4(1-y)^2\biggl(\mathrm{Li}_2(1-y) + \ln^2 y + \frac{1}{2}\ln^2(1-y) \\ &\quad -2\zeta_2\biggr) + (6-4y)\ln y + \biggl(y - \frac{3}{2}\biggr)\ln(1-y) + \frac{9}{4} - 2y\biggr] + \frac{1}{3}\frac{\alpha}{2\pi}L\delta^{(1)LL}_{\bar{\nu} CC}.\end{aligned} \tag{39}$$

Argument of the large logarithm depends on the energy scale of the process under consideration and the muon mass. We have two scales: $\hat{s}$ and $\hat{Q}^2$. In the actual calculations the logarithm with the muon mass singularity arises in the form $L = \ln(\hat{s}/m_l^2)$. So we choose $\hat{s}$, while the difference between the two possibilities appears in our case only in terms of the order $\mathcal{O}\,(\alpha^2 L)$, which are omitted now in any case.

Using the formalism of the fragmentation function approach [22] we can get the next–to–leading corrections [3] of the order $\mathcal{O}\,(\alpha^2 L)$. But, as will be seen from our numerical estimates below, they are small compared with the requested precision tag.

4 Numerical Results and Conclusions

Summing up all the different RC contributions considered above, and applying the $\overline{\mathrm{MS}}$ subtraction of the initial state quark mass singularity we arrive at the result for the corrections to neutrino–quark cross sections:

$$\frac{\mathrm{d}^2\sigma_i^{\mathrm{Corr.}}}{\mathrm{d}x\;\mathrm{d}y} = \frac{\mathrm{d}^2\sigma_i^{\mathrm{Born}}}{\mathrm{d}x\;\mathrm{d}y}(1 + \delta_i + \delta_i^{(2)LL} - \delta_{\overline{\mathrm{MS}}}), \tag{40}$$

where $\delta_i^{(2)LL}$ vanishes in the NC case.

For neutrino–quark CC scattering we have

$$\begin{aligned}
\delta_{\nu CC} = \frac{\alpha}{\pi}\Big\{ & -\frac{3}{2}\ln\frac{\hat{s}}{M_Z^2} + \Big(\frac{3}{4} - \frac{1}{2}y - \frac{1}{2}\ln(1-y) + \ln y\Big)\ln\frac{\hat{s}}{m_l^2} \\
& + \frac{1}{2}\ln(1-y)\ln y - \frac{3}{2}\mathrm{Li}_2\,(y) - \frac{1}{2}\ln^2 y - \frac{1}{2}\ln^2(1-y) \\
& + (1-y)\ln(1-y) - \frac{7}{4}\ln y + \frac{1}{2}y\ln y + \frac{5}{4}y + \frac{1}{2} + 2\zeta_2 \\
& + Q_1\Big(3 - \frac{3}{2}\ln\frac{\hat{s}}{M_Z^2} + \zeta_2 - \ln(1-y)\ln y - 2\mathrm{Li}_2\,(y)\Big) \\
& + Q_1^2\Big(-\frac{2}{3}\ln\frac{\hat{Q}^2}{m_1^2} + \frac{23}{72} - \frac{5}{12}y + \frac{1}{24}y^2 - \zeta_2\Big)\Big\},
\end{aligned} \tag{41}$$

where we used the charge conservation law $Q_1 = Q_2 + Q_l$ and the explicit value $Q_l = -1$. For antineutrino–quark CC scattering we have

$$\begin{aligned}
\delta_{\bar{\nu}\,CC} = \frac{1}{(1-y)^2}\,\frac{\alpha}{\pi}\Big\{ & \Big((1-y)^2\ln\frac{y}{1-y} - \frac{y}{2} + \frac{3}{4}\Big)\ln\frac{\hat{s}}{m_l^2} + (1-y)^2\Big(2\zeta_2 - \mathrm{Li}_2\,(y) \\
& + \ln(1-y)\ln y - \frac{1}{2}\ln^2 y - \ln^2(1-y)\Big) + \Big(-\frac{7}{4} + \frac{5}{2}y - y^2\Big)\ln\frac{y}{1-y}
\end{aligned}$$

[3] See Ref. [23] for an analogous calculation.

$$
- \frac{5}{4} + y + Q_1\Big[\frac{1}{2} - \frac{5}{2}y + \frac{7}{4}y^2 - \frac{3}{2}(1-y)^2 \ln \frac{\hat{s}(1-y)}{M_Z^2}
+ (1-y)^2\Big(2\mathrm{Li}_2\,(y) + \ln(1-y)\ln y - \zeta_2\Big)\Big] + Q_1^2\Big[-\frac{2}{3}(1-y)^2 \ln \frac{\hat{Q}^2}{m_1^2}
- (1-y)^2\zeta_2 - \frac{1}{18}y^2 - \frac{2}{9}y + \frac{23}{72}\Big]\Big\}. \tag{42}
$$

For the case of NC neutrino scattering we get

$$
\delta_{\nu NC} = \frac{1}{g_L^2 + g_R^2(1-y)^2}\Big\{\frac{\alpha}{\pi}Q_1^2\Big[g_R^2\Big(-\frac{2}{3}(1-y)^2\ln\frac{\hat{Q}^2}{m_1^2} - \frac{1}{18}(1-y)^2 + \frac{1}{3}(1-y)
+ \frac{1}{24} - \zeta_2(1-y)^2\Big) + g_L^2\Big(-\frac{2}{3}\ln\frac{\hat{Q}^2}{m_1^2} + \frac{1}{24}(1-y)^2 + \frac{1}{3}(1-y)
- \frac{1}{18} - \zeta_2\Big)\Big] + \tilde{g}_L^2 + \tilde{g}_R^2(1-y)^2\Big\} - 1. \tag{43}
$$

The expression for the one–loop radiative correction to the antineutrino NC process can be received from the above one using the substitution:

$$
\delta_{\bar{\nu}NC} = \delta_{\nu NC}(g_L \leftrightarrow g_R, \quad \tilde{g}_L \leftrightarrow \tilde{g}_R). \tag{44}
$$

Formulae (41,42,43) completely agree with the ones derived in Ref. [4]. Moreover, our formulae for the CC scattering case agree with the calculations presented in Ref. [24], where the explicit expressions were given for νd and $\bar{\nu}u$ CC scattering channels. For the comparison, one should subtract from their formulae for $g^{\nu}(y,S)$ and $g^{\bar{\nu}}(y,S)$ the quantities C and $(1-y)^2C$, respectively,

$$
C = \ln\frac{M_W^3}{\mu M_Z^2} + \frac{1}{4}. \tag{45}
$$

The subtraction reflects the choice of the $\overline{\mathrm{MS}}$ renormalization scheme.

Let us consider numerical results obtained for the following conditions: the fixed neutrino energy $E_\nu = 80$ GeV; isoscalar nuclear target; cut on the energy of the final state hadronic system $\hat{E}_{\mathrm{hadr}} \geq 10$ GeV. We used the CTEQ4L set [25] of parton density functions. In Table 1 we present numerical values and the absolute shifts due to radiative corrections of the quantities [26], constructed from the cross sections of neutrino DIS,

$$
R^\nu = \frac{\sigma_{NC}^\nu(\nu_\mu N \to \nu_\mu X)}{\sigma_{CC}^\nu(\nu_\mu N \to \mu^- X)}, \tag{46}
$$

(ISCH,ILLA), EW scheme	R^ν	δR^ν_{NC}	δR^ν_{CC}	$\Delta^\nu \sin^2\theta_W$	R^-	$\Delta^- \sin^2\theta_W$
(0,1), G_F	0.31006	−0.00291	−0.02147	−0.01130	0.27094	−0.00737
(1,1), G_F	0.31063	0.00071	−0.02327	−0.01044	0.27195	−0.00636
(1,2), G_F	0.31067	0.00071	−0.02315	−0.01039	0.27196	−0.00637
(1,2), $\alpha(0)$	0.31080	−0.05816	0.03743	−0.01020	0.27209	−0.00623

Table 1
Effect of RC on R^ν, R^-, and $\sin^2\theta_W$ values in different approximations.

$$R^- = \frac{\sigma^\nu_{NC}(\nu_\mu N \to \nu_\mu X) - \sigma^{\bar\nu}_{NC}(\bar\nu_\mu N \to \bar\nu_\mu X)}{\sigma^\nu_{CC}(\nu_\mu N \to \mu^- X) - \sigma^{\bar\nu}_{CC}(\bar\nu_\mu N \to \mu^+ X)}, \tag{47}$$

$$\delta R^\nu_{NC} = \frac{\sigma^{\mathrm{Corr.}}_{\nu NC} - \sigma^{\mathrm{Born}}_{\nu NC}}{\sigma^{\mathrm{Born}}_{\nu NC}}, \qquad \delta R^\nu_{CC} = -\frac{\sigma^{\mathrm{Corr.}}_{\nu CC} - \sigma^{\mathrm{Born}}_{\nu CC}}{\sigma^{\mathrm{Born}}_{\nu CC}}. \tag{48}$$

The Born level values are $R^\nu_0 = 0.31764$ and $R^-_0 = 0.27831$. In the computations we used the following values of constants and parameters taken from [27] (the same as the ones given by Eq. (4.6) of Ref. [15]). Flag `ISCH` defines the choice of the initial quark mass singularity treatment: `ISCH`=0 means no any subtraction (as has been done in Ref. [4]), and `ISCH`=1 corresponds to the $\overline{\mathrm{MS}}$ subtraction scheme as discussed above. The order of the series in the leading large logarithms with the lepton mass singularity is governed by flag `ILLA`: $O(\alpha L)$ for `ILLA`=1 and $O(\alpha^2 L^2)$ for `ILLA`=2.

For the illustrations we used the simple tree–level relations between the shifts of $R^{\nu,-}$ and $\sin^2\theta_W$:

$$\delta R^\nu = \left(1 - \frac{40}{27}\sin^2\theta_W\right)\Delta^\nu \sin^2\theta_W, \qquad \delta R^- = \Delta^- \sin^2\theta_W, \tag{49}$$

where δR^ν and δR^- are the differences between the corrected quantities and the tree–level ones. In a real case of experimental data analysis the contribution of radiative corrections should be estimated within a general fit including all other effects like detector efficiencies, nuclear shadowing *etc.*

One can see by comparing the first two lines in Table 1, that the $\overline{\mathrm{MS}}$ subtraction of the initial quark mass singularity gives a numerically important contribution. We considered two electroweak schemes: the G_{Fermi} and $\alpha(0)$ (see Ref. [15] and references therein for the definition). We found that the EW scheme dependence is visible, but less than the one observed in Ref. [15].

One can compare our numbers with the corresponding results given in Table 1 of Ref. [15] for the case of the energy cut $E^{\mathrm{LAB}}_{\mathrm{had+phot}} > 10$ GeV. The difference between the two calculations appeared to be not small. Partially this is related to different treatments of the effect of the scaling violation in the parton density functions. A more detailed comparison is in progress and will be published elsewhere.

In Figures 1,2 we present the relative value of the sum of radiative corrections,

$$\delta_i + \delta_i^{(2)LL} - \delta_{\overline{\mathrm{MS}}}),$$

for two channels (neutrino–up quark NC scattering and neutrino–down quark CC scattering) as a function of y for three fixed values of x. The neutrino beam energy and other parameters are the same as the ones defined for Table 1. One can see that the behavior of the corrections is rather smooth. Note that in the case of a different choice of variables, when Q^2 is defined from the observation of the outgoing muon, the size of the corrections for the CC case becomes several times larger[4].

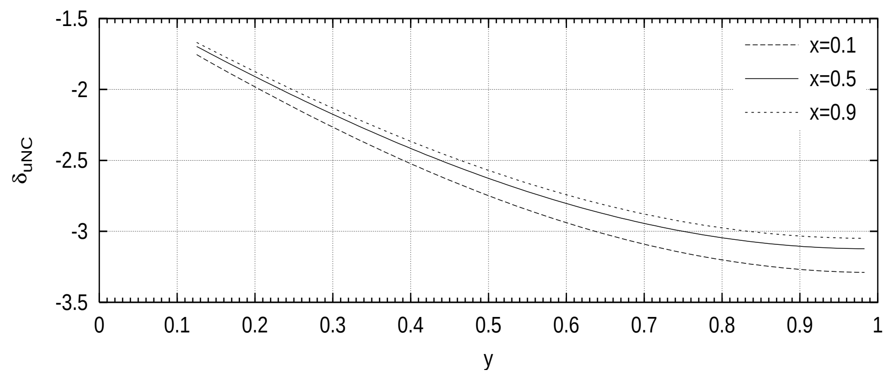

Fig. 1. Relative effect of radiative corrections to $\nu - u$ NC scattering as a function of y for three fixed values of x.

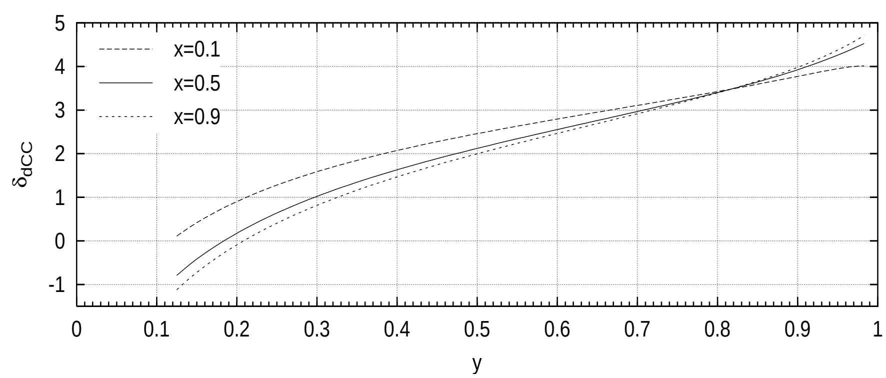

Fig. 2. Relative effect of radiative corrections to $\nu - d$ CC scattering as a function of y for three fixed values of x.

Let us consider the sources of the theoretical uncertainty, related to the incomplete knowledge of the radiative corrections. The sources are listed in Table 2. First, we have observed already the electroweak scheme dependence. From the general point of view, the G_{Fermi} scheme looks preferable for our problem. By trying the other EW scheme, we get an estimate of how large can be unknown higher order electroweak corrections.

[4] The same effect is well known in the conventional charged lepton deep inelastic scattering.

Source of uncertainty	Estimated value		
	for R^ν	for $\Delta^\nu \sin^2\theta_W$	for R^-
higher order EW RC	$1.5 \cdot 10^{-4}$	$2.2 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$
higher order QED RC	$0.5 \cdot 10^{-4}$	$0.7 \cdot 10^{-4}$	$0.2 \cdot 10^{-4}$
$\mathcal{O}(\alpha\alpha_s)$	$2.2 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$
all combined	$2.7 \cdot 10^{-4}$	$4.0 \cdot 10^{-4}$	$2.0 \cdot 10^{-4}$

Table 2
Estimates of different contributions to the theoretical uncertainty.

The pure QED corrections [5] are in our case not large, and by looking at the leading second order logarithmic contribution, we can put a limit on unknown higher order QED contributions. As the main source of the theoretical uncertainty we consider the contribution of the radiative corrections of the order $\mathcal{O}(\alpha\alpha_s)$. They can appear as in the loop insertions into the W-boson propagator as well as in amplitudes where photonic and gluonic lines appear simultaneously. In our approach, we separated the EW RC from the QCD ones. After all, we receive a direct product of QCD effects (see i.e. Ref. [28]) and EW corrections, which doesn't cover the full $\mathcal{O}(\alpha\alpha_s)$ answer. Moreover, as can be seen from our calculations, we always consider $\tilde{Q}^2$ to be the argument of the partonic density function. But in the case of hard photon radiation off lepton, $\tilde{Q}^2$ doesn't coincide with the square of the "true" hadronic momentum transfer. This leads to a certain effect in $\mathcal{O}(\alpha\alpha_s)$. To estimate the corresponding uncertainty we varied the value of Q^2 in the argument of the PDFs. As the result of our estimates we found that our result for radiative corrections can receive up to $\pm 3\%$ of a relative shift. Having a unique value of uncertainty for the functions of two variables seems to be reasonable, because the functions are relatively flat, and because, in the derivation of the number we looked both at the variations at the differential level and at the ones for the integrated cross sections. Note that we estimated the uncertainties in the form of maximal variations. A work is going on to extend our consideration to the case of other variables and to avoid the effect of improper argument in PDFs, mentioned above.

It is worth to note, that radiative corrections to the same series of processes calculated in a different set of variables can have a completely different behavior. For example, in the variables adopted by the NuTeV experiment, the size of corrections to the CC scattering (where the energy and angle of the outgoing charge lepton can be measured) are much bigger than the corrections presented here. In the same manner, estimates of the theoretical uncertainties should be considered taking into account the choice of variables and, may be, other relevant experimental conditions. Nevertheless we agree with the authors of Ref. [15], who claimed that the higher order contributions to the theoretical uncertainty seem to have been underestimated by the NuTeV experiment [2].

Our analytical results are combined into a `FORTRAN` code, which can be directly used

[5] Unambiguous separation of EW and QED corrections in the CC channel is not possible.

for experimental data analysis. As can be seen from the numerical estimates the effect of radiative corrections is greater than the present experimental uncertainty, see *e.g.* the NuTeV [2] result:

$$\sin^2\theta_W = 0.2277 \pm 0.0013(\text{stat.}) \pm 0.0009(syst.). \tag{50}$$

That makes it important to take RC into account in the proper way. Estimates of the theoretical uncertainty becomes also relevant for the derivation of the resulting error in the analysis of experimental data in the present and future precision experiments on neutrino DIS.

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Appendix A
List of Integrals for Soft Photon RC

All relevant integrals are defined in the reference frame $\vec{p}_2 = 0$. They are:

$$\int \frac{\mathrm{d}^3k}{\omega}\frac{p_1^2}{(p_1k)^2} = 4\pi\left(\ln\frac{2\bar\omega}{\lambda} - \frac{1}{2}L_1\right), \tag{A.1}$$

$$\int \frac{\mathrm{d}^3k}{\omega}\frac{p_2^2}{(p_2k)^2} = 4\pi\left(\ln\frac{2\bar\omega}{\lambda} - 1\right), \tag{A.2}$$

$$\int \frac{\mathrm{d}^3k}{\omega}\frac{k_2^2}{(k_2k)^2} = 4\pi\left(\ln\frac{2\bar\omega}{\lambda} - \frac{1}{2}L_l\right), \tag{A.3}$$

$$\int \frac{\mathrm{d}^3k}{\omega}\frac{2p_1p_2}{(p_1k)(p_2k)} = 2\pi\left(2\ln\frac{2\bar\omega}{\lambda}L_1 - \frac{1}{2}L_1^2 - 2\zeta(2)\right), \tag{A.4}$$

$$\int \frac{\mathrm{d}^3k}{\omega}\frac{2p_2k_2}{(p_2k)(k_2k)} = 2\pi\left(2\ln\frac{2\bar\omega}{\lambda}L_l - \frac{1}{2}L_l^2 - 2\zeta(2)\right), \tag{A.5}$$

$$\begin{aligned}\int \frac{\mathrm{d}^3k}{\omega}\frac{2p_1k_2}{(p_1k)(k_2k)} &= 2\pi\Bigg[2\ln\frac{2\bar\omega}{\lambda}\left(\ln\frac{w}{m_l^2} + \ln\frac{w}{m_1^2}\right) - \ln\frac{w}{m_1^2}L_1 - \ln\frac{w}{m_l^2}L_l \\ &+\frac{1}{2}\ln^2\frac{w}{m_1^2} + \frac{1}{2}\ln^2\frac{w}{m_l^2} - \ln^2\frac{\hat{Q}^2}{\hat{s}} - 4\zeta(2) + 2\mathrm{Li}_2\left(\frac{1+c}{2}\right)\Bigg],\end{aligned} \tag{A.6}$$

$$
\begin{aligned}
&L_1 = \ln\frac{4(p_1^0)^2}{m_1^2} = \ln\frac{\hat{Q}^2}{m_1^2} + \ln\frac{\hat{Q}^2}{m_2^2}, \qquad L_l = \ln\frac{4(k_2^0)^2}{m_l^2} = \ln\frac{\hat{s}^2}{m_l^2 m_2^2},\\
&w = 2p_1k_2 \approx \hat{s} - \hat{Q}^2, \qquad \hat{Q}^2 = 2m_2p_1^0,\\
&\frac{1+c}{2} \equiv \frac{1+\cos(\widehat{\vec{p}_1\vec{k}_2})}{2} = 1 - \frac{m_2^2(\hat{s}-\hat{Q}^2)}{\hat{s}\hat{Q}^2} \approx 1.
\end{aligned}
$$

Terms which are suppressed by the ratio of a quark or muon mass to a large energy scale (beam energy or momentum transferred) are neglected.

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