ELECTROWEAK RADIATIVE CORRECTIONS TO NEUTRINO DIS

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Radiative corrections to neutrino deep inelastic scattering are revisited. One-loop electroweak corrections are re-calculated within the automatic SANC system. Terms with mass singularities are treated including higher order leading logarithmic contributions. The results are suited to be used in the data analysis of the NOMAD experiment. The present theoretical accuracy in description of the process is discussed.

Modern experiments such as NOMAD [1], NuTeV [2], and CHORUS [3] made a serious step forward in studies of neutrino deep inelastic scattering. Their precision measurements made it necessary to update the accuracy level of the theoretical description of the process and therefore to revisit the calculations of radiative corrections (RC).

Our study is motivated by the request from the NOMAD experiment. Electroweak (EW) radiative corrections to neutrino-nucleon scattering should have been implemented into the general Monte Carlo system for the experimental data analysis. Certain experimental conditions of particle registration and event selection should have been taken into account. In order to make the relevant subroutine describing the corrections fast, we had to look for an analytical answer.

For the calculation we used the modern technique of automatic calculations within the SANC system [4,5] developed for Support of Analytic and Numeric calculations for experiments at Colliders. Besides the one-loop calculation, we consider certain contributions of higher orders and discuss the theoretical uncertainty due to unknown electroweak corrections for the case of the concrete experimental study.

We will consider the process of deep inelastic scattering (DIS) in the framework of the quark-parton model, assuming that we are in the proper kinematical region. For our analytical calculations it is important to fix the choice of kinematical variables:

$$y = \frac{\tilde{Q}^2}{\hat{s}}, \qquad \tilde{Q}^2 = -2p_1(p_2+k), \qquad x = \frac{\tilde{Q}^2}{yS},$$
$$\hat{s} = (k_1+p_1)^2 \approx xS, \qquad S = (k_1+P)^2, \tag{1}$$

where $p_{1(2)}$ and $k_{1(2)}$, are the four-momenta of the incoming (outgoing) quark and the incoming (outgoing) lepton, respectively. The photon momentum, k, should be set to zero for non-radiative processes. P is the initial nucleon momentum. We will use $m_{1,2,l}$ ($Q_{1,2,l}$) for the masses (charges) of the initial quark, the final quark, and the muon.

The radiatively corrected neutrino DIS cross section can be represented as the sum of the Born distribution with the contributions due to virtual loop diagrams (Virt), soft photon emission (Soft), and hard photon emission (Hard). We assume that the momentum transfer square is small compared with the W-boson mass: corrections of the order $\alpha\tilde{Q}^2/M_W^2$ are omitted.

Contributions due to virtual EW corrections are re-calculated by means of the SANC system. All the contributing one-loop diagrams were considered in the frame of the Standard Model. The infrared singularity was regularized by introducing an auxiliary photon mass λ. Emission of a soft photon in neutrino DIS was described in the standard way by the accompanying radiation factors. The soft photon energy, ω, was limited by the parameter $\bar{\omega}$. We considered the problem in the rest reference frame of the final quark, $\vec{p}_2 = 0$, which is equivalent (in the soft photon limit) to the R-reference frame (where $\vec{k}+\vec{p}_2 = 0$) used in the calculation of the hard photon contribution.

In the sum of the soft and hard photonic corrections, the auxiliary parameter $\bar{\omega}$ cancels out. The infrared singular terms (with logarithm of λ) cancel out in the sum of the virtual and soft contributions. Moreover in the total sum, the large logarithms (mass singularities) with the final state quark mass m_2 disappear in accordance with the Kinoshita-Lee-Nauenberg theorem.

Large logarithms containing the initial quark mass, $\ln(\tilde{Q}^2/m_1^2)$, remain in the sum of all the contributions. But these logs have been already effectively taken into account in the parton density functions (PDFs), where one has an effective coupling constant $\alpha_s^{\rm eff} \approx \alpha_s + \alpha Q_1^2/C_F$, with α_s and C_F being the strong coupling constant and the QCD color factor. The nontrivial difference between the QED evolution and the QCD one starts to appear from higher orders, and the corresponding numerical effect is small compared to the remaining QCD uncertainties in PDFs [6]. Using the initial condition for the non-singlet NLO QED quark structure function in the $\overline{\rm MS}$ scheme[a], we get the terms to be subtracted from the full calculation with massive quarks:

$$\delta_{\overline{\rm MS}} = Q_1^2 \frac{\alpha}{2\pi}\left(-\frac{4}{3}\ln\frac{\tilde{Q}^2}{m_1^2} - \frac{17}{9}\right). \tag{2}$$

In the case of neutrino CC scattering, the large logarithms singular in the limit $m_l \to 0$ remain in the final answer. These terms are in agreement with

[a] A similar consideration can be performed in the DIS and other schemes as well.

the prediction of the renormalization group approach. They can be described by the electron (muon) fragmentation (structure) function approach. For our purposes it is enough to consider only the leading log approximation in the $\mathcal{O}\left(\alpha L\right)$ and $\mathcal{O}\left(\alpha^2 L^2\right)$ orders including the contribution of electron-positron pairs. Convolution with the leading order splitting function gives the first order logarithmic correction,

$$\delta^{(1)LL}_{\nu CC} = \frac{\alpha}{2\pi} L \biggl(2\ln y - \ln(1-y) + \frac{3}{2} - y \biggr). \tag{3}$$

The above expression reproduce the the leading log part of the complete result for the $\mathcal{O}\left(\alpha\right)$ correction. By convolution with the $\mathcal{O}\left(\alpha^2\right)$ terms of the fragmentation function, we get the second order leading log corrections to the CC neutrino DIS,

$$\begin{aligned}\delta^{(2)LL}_{\nu CC} &= \frac{1}{2}\left(\frac{\alpha}{2\pi}L\right)^2 \biggl[4\ln^2 y - 4\ln y\ln(1-y) + \frac{1}{2}\ln^2(1-y) - 4\zeta_2 \\ &+ (6-4y)\ln y + (3y-4)\ln(1-y) + \frac{9}{4}\biggr] + \frac{1}{3}\,\frac{\alpha}{2\pi}L\delta^{(1)LL}_{\nu CC}. \end{aligned} \tag{4}$$

Similar analytical formulae can be obtained for the antineutrino CC scattering case. Argument of the large logarithm depends on the energy scale of the process under consideration and the muon mass. We choose $\hat{s}$ as the energy scale of the logarithm $L = \ln(\hat{s}/m_l^2)$, since this argument appeared in the full one-loop calculation.

Summing up all the different RC contributions considered above, and applying the $\overline{\mathrm{MS}}$ subtraction of the initial state quark mass singularity we arrive at the result for the corrections to neutrino-quark cross sections:

$$\frac{\mathrm{d}^2\sigma_i^{\mathrm{Corr.}}}{\mathrm{d}x\;\mathrm{d}y} = \frac{\mathrm{d}^2\sigma_i^{\mathrm{Born}}}{\mathrm{d}x\;\mathrm{d}y}(1 + \delta_i + \delta_i^{(2)LL} - \delta_{\overline{\mathrm{MS}}}), \tag{5}$$

where the index i denotes the type of the process under consideration ($\nu q\; CC$, $\bar{\nu}q\; NC$ and so on). Quantity $\delta_i^{(2)LL}$ vanishes in the NC case. Explicit analytical formulae for all the contributions can be found in our preprint [7]. Formulae for the first order corrections δ_i completely agree with the ones derived in Ref. [8].

Let us consider numerical results obtained for the following conditions: the fixed neutrino energy $E_\nu = 80$ GeV; isoscalar nuclear target; cut on the energy of the final state hadronic system $\hat{E}_{\mathrm{hadr}} \geq 10$ GeV. We used the CTEQ4L set [9] of parton density functions. In Table 1 we present numerical values and

(ISCH,ILLA), EW scheme	R^ν	δR^ν_{NC}	δR^ν_{CC}	$\Delta^\nu \sin^2\theta_W$
(0,1), G_F	0.31006	-0.00291	-0.02147	-0.01130
(1,1), G_F	0.31063	0.00071	-0.02327	-0.01044
(1,2), G_F	0.31067	0.00071	-0.02315	-0.01039
(1,2), $\alpha(0)$	0.31080	-0.05816	0.03743	-0.01020

Table 1: Effect of RC on R^ν and $\sin^2\theta_W$ values in different approximations.

the absolute shifts due to radiative corrections of the quantities

$$R^\nu = \frac{\sigma^\nu_{NC}(\nu_\mu N \to \nu_\mu X)}{\sigma^\nu_{CC}(\nu_\mu N \to \mu^- X)}, \tag{6}$$
$$\delta R^\nu_{NC} = \frac{\sigma^{\text{Corr.}}_{\nu NC} - \sigma^{\text{Born}}_{\nu NC}}{\sigma^{\text{Born}}_{\nu NC}}, \qquad \delta R^\nu_{CC} = -\frac{\sigma^{\text{Corr.}}_{\nu CC} - \sigma^{\text{Born}}_{\nu CC}}{\sigma^{\text{Born}}_{\nu CC}}.$$

The Born level values is $R^\nu_0 = 0.31764$. In the computations we used the following values of constants and parameters taken from [10]. Flag `ISCH` defines the choice of the initial quark mass singularity treatment: `ISCH`=0 means no any subtraction (as has been done in [8]), and `ISCH`=1 corresponds to the $\overline{\text{MS}}$ subtraction scheme as discussed above. The order of the series in the leading large logarithms with the lepton mass singularity is governed by flag `ILLA`: $O(\alpha L)$ for `ILLA`=1 and $O(\alpha^2 L^2)$ for `ILLA`=2.

For the illustrations we used the simple tree-level relation between the shifts of R^ν and $\sin^2\theta_W$:

$$\delta R^\nu = \left(1 - \frac{40}{27}\sin^2\theta_W\right)\Delta^\nu \sin^2\theta_W. \tag{7}$$

where δR^ν are the differences between the corrected quantities and the tree-level ones. In a real case of experimental data analysis the contribution of radiative corrections should be estimated within a general fit including all other effects like detector efficiencies, nuclear shadowing *etc.* One can see that the $\overline{\text{MS}}$ subtraction of the initial quark mass singularity gives a numerically important contribution. We considered two electroweak schemes: the G_{Fermi} and $\alpha(0)$ (see Ref. [11] and references therein). One can compare our numbers with the corresponding results given in Table 1 of [11] for the case of the energy cut $E^{\text{LAB}}_{\text{had+phot}} > 10$ GeV. The difference between the two calculations is related partially to different treatments of the effect of the scaling violation in the parton density functions. A more detailed comparison is in progress and will be published elsewhere.

The main source of the theoretical uncertainty is the contribution of the radiative corrections of the order $\mathcal{O}(\alpha\alpha_s)$. Unknown higher order electroweak corrections are also relevant, as one can see from the results obtained in different EW schemes. On the other hand we don't expect any considerable enhancement factor in the unknown contributions. So, the estimate of the uncertainty can be done starting from the present calculations. We put the upper estimate the part of the theoretical uncertainty in the description of neutrino-quark cross section coming from the EW corrections to be of the order 0.3%, which corresponds to the uncertainty in $\Delta^\nu \sin^2\theta_W$ of the order ± 0.0004 (in absolute units). We agree with the authors of [11] who claimed that the higher order contributions to theoretical uncertainty have been underestimated by the NuTeV experiment [2] Our analytical results are combined into a `FORTRAN` code, which can be directly used for experimental data analysis.

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