QED Radiative Corrections within the SANC project

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An automatic calculation of the QED radiative corrections in the framework of the SANC computer system is described. A collection of the computer programs written in FORM3 language is aimed at the creation of a database of analytic results to be used for a theoretical support of experiments at high energy accelerators.

Here the scheme of an automatic analytical calculation of the QED radiative corrections to the fermionic decays of the Z, H and W boson in the framework of the SANC system is presented.

1. ITNRODUCTION

A computer system SANC for calculation of the one-loop corrections performs:

1. Analytic calculations by `FORM3` [1];
2. Numeric calculations by `Fortran`.

In this talk I consider the particular example – the analytic calculations of the QED part of the radiative corrections for Z (W, H) boson decays to fermion pairs. More extensive explanations of this computer system is given in the talk of D. Bardin at this conference [2].

2. THE BASIC PROCEDURES NEEDED

SANC [3] contains some set of `FORM3` programs and procedures needed for calculation the radiative corrections at one-loop level for any process. Some of the main procedures are:

`FeynmanRules.prc` – substitutes different **vert** and **pr** functions for vertices and propagators.

`GammaRight.prc` – moves the γ_5 to right in the expression of a given amplitude. `Pulling.prc` and `Pullingc.prc` – moves momenta $\hat{p}_i$ of the external fermions close to corresponding bispinors.

`Diraceq.prc` and `Diraceqc.prc` – realizes corresponding Dirac equations for bispinors.

All the renormalization constants are calculated in the programs `CalcFerRenConst.frm` and `CalcBosRenConst.frm` using the procedures `FerRenConst.prc` and `UniBosRenConst.prc` which prepare the one-loop counter terms.

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3. ONE-LOOP FORM FACTORS OF THE PROCESSES

The program `vff_ve_rxi.frm` computes the one-loop amplitudes of the decays of Z (W, H) bosons with momentum Q to fermions with momenta k_1 and k_2. In the programs `Ren_Z_decay.frm` or `Ren_W_decay.frm` or `Ren_H_decay.frm` they meet counter terms and as the result we obtain the corresponding renormalised amplitudes in terms of scalar form factors (SFF). The QED parts of the one-loop renormalised amplitudes obtained in this way are:

$$\begin{aligned}
M^{1-\rm loop}_{Zff} &= \frac{ig}{2\cos\theta_W}\,\epsilon^z_\mu\,\frac{g^2}{16\pi^2} \\
&\times\bar{u}(k_1)\Big(\,i\,I^{(3)}_f\;\gamma_\mu\,(1+\gamma_5)\;\mathbf{F}^{\mathbf{Z}}_{\mathbf{L}} \\
&-2\,i\,Q_f\,\sin^2\theta_W\;\gamma_\mu\;\mathbf{F}^{\mathbf{Z}}_{\mathbf{Q}} \\
&+I^{(3)}_f m_f\left(k_{2\,\mu}-k_{1\,\mu}\right)\,\mathbf{F}^{\mathbf{Z}}_{\mathbf{D}}\Big)\,v(k_2), \\
M^{1-\rm loop}_{Wff} &= \frac{ig}{2\sqrt{2}}\,\epsilon^W_\mu\,\frac{g^2}{16\pi^2} \\
&\times\bar{u}(k_1)\,i\,\gamma_\mu\,(1+\gamma_5)\;\mathbf{F}^{\mathbf{W}}_{\mathbf{L}}\,v(k_2), \\
M^{1-\rm loop}_{Hff} &= -\frac{igm_f}{2M_W}\,\frac{g^2}{16\pi^2}\,\bar{u}(k_1)\,\mathbf{F}^{\mathbf{H}}_{\mathbf{S}}\;v(k_2),
\end{aligned}$$

where only the QED parts of the SFF are taken into account.

The interference of these amplitudes with the corresponding Born amplitudes gives rise to the virtual QED contribution in the radiative corrections to the decay widths. It is obtained in the procedure `BffdecayQED.prc`.

4. SOFT BREMSSTRAHLUNG

The other contribution to the QED radiative corrections, also infrared divergent, comes from the real photon emission from the charged legs of the tree diagrams. Two programs, starting from the procedure `BremBff.prc`, calculate the bremsstrahlung processes $B \longrightarrow f + \bar{f} + \gamma$:

`NC_decay_Brem.frm` for neutral boson decays, $B = Z, H$;

`W_decay_Brem.frm` for charged boson decay, $B = W^+, W^-$.

The soft bremsstrahlung is separated in the procedure `zeroSoftBff.prc` taking into account that the soft photon energy lies in the small interval $0 \leq p^0 \leq \omega$. Then a proof is made that the soft amplitude is proportional to the Born one. The corresponding expressions for Z, H, and W decays are:

$$M^{\rm Soft}_{Z(H)ff} = e\, Q_f \left(\frac{k_1 \cdot \epsilon^\gamma}{k_1 \cdot p} - \frac{k_2 \cdot \epsilon^\gamma}{k_2 \cdot p} \right) M^{\rm Born}_{Z(H)ff},$$

$$M^{\rm Soft}_{Wff} = e \left(Q_u \frac{k_1 \cdot \epsilon^\gamma}{k_1 \cdot p} - Q_d \frac{k_2 \cdot \epsilon^\gamma}{k_2 \cdot p} - Q_W \frac{(k_1 + k_2) \cdot \epsilon^\gamma}{(k_1 + k_2) \cdot p} \right) M^{\rm Born}_{Wff}.$$

After we put the real photon momentum in the form $p = p^0 n$, we have $\vec{n} \cdot \vec{k}_2 = |\vec{k}_2| \cos\theta$ and then

$$M^{\rm Soft} = \frac{1}{p^0} F(k_1, k_2, n)\, M^{\rm Born}.$$

Then programs carry out an integration of the modulus squared amplitudes $M^{\rm Soft}$ over photon phase space with small radius ω to obtain the soft bremsstrahlung contribution to the decay widths:

$$\Gamma^{\rm Soft} = \Gamma^{\rm Born} \int\limits_{m_\gamma}^{\omega} \frac{(p^0)^2 dp^0}{(2\pi)^2 2p^0} \int\limits_{-1}^{1} \frac{d\cos\theta}{(p^0)^2} F^2(k_1, k_2, n).$$

The integration over p^0 gives the well known infrared divergence in terms of $\ln\frac{2\omega}{m_\gamma}$. The integration over $\cos\theta$ is realized in the procedures `NCdecaySoft.prc` and `WdecaySoft.prc`. So we obtain $\Gamma^{\rm Soft}_{Z(H)ff}$ and $\Gamma^{\rm Soft}_{Wff}$.

5. HARD BREMSSTRAHLUNG

The calculation of the hard bremsstrahlung is more complicated. We have bremsstrahlung kinematics with a new variable $s' = -(k_1 + k_2)^2$ different from $s = M^2 = -(k_1 + k_2 + p)^2$. After the bremsstrahlung amplitude is squared in the procedure `BffHardSection.prc`, the two above mentioned programs perform integration over the whole phase space of the three outgoing particles. The integration over $\cos\theta$ is realized in the procedures `NCdecayHard.prc` and `WdecayHard.prc` using two different tables of integrals. The integration over s' is done in the procedures `NCdecayHardInt.prc` and `WdecayHardInt.prc` with tables of integrals. The analytical results are lengthy and it is impossible to show them here.

6. TOTAL WIDTHS

In the procedures `NCdecayTotalQED.prc` and `NCdecayTotalQED.prc` all the QED radiative correction contributions to the fermionic widths of boson decays are put together as follows:

$$\Gamma^{\rm QED} = \Gamma^{\rm Virt} + \Gamma^{\rm Soft} + \Gamma^{\rm Hard}.$$

7. RESULTS

From the output files `NC_decay_Brem.log` and `W_decay_Brem.log` the analytical results go to the software product `s2n.f` (symbols–to–numbers), where the Fortran codes for numeric calculation of bosons decay widths are being created.

The description of computer system SANC (working environment of procedures, user guides and etc) and of the bank comprising analytic results and histogramms is in preparation [3].

REFERENCES

1. J. A. M. Vermaseren, “New features of FORM”, math-ph/0010025.
2. D.Yu. Bardin et al., “SANC press release”, these Proceedings.
3. A. Andonov et al., a write-up in preparation.