SANC: $2 \to 2$ PROCESSES

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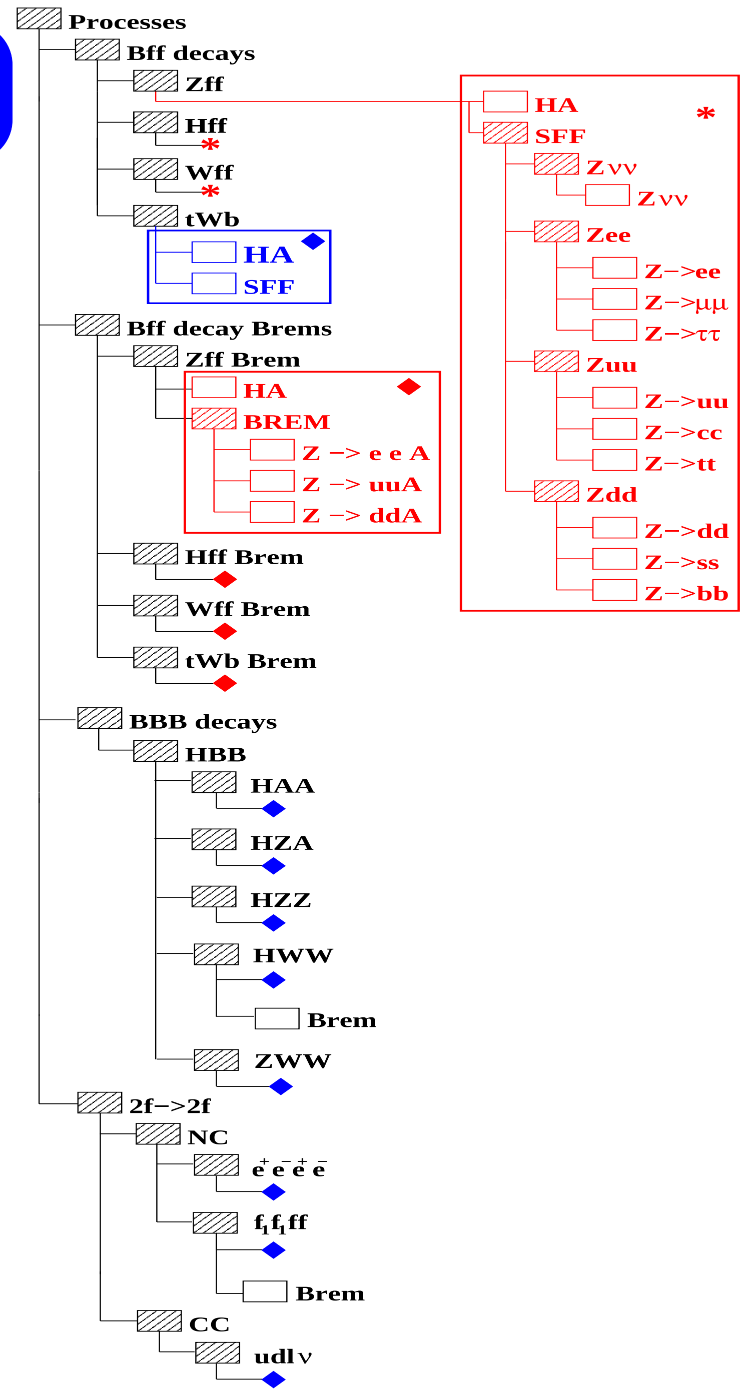
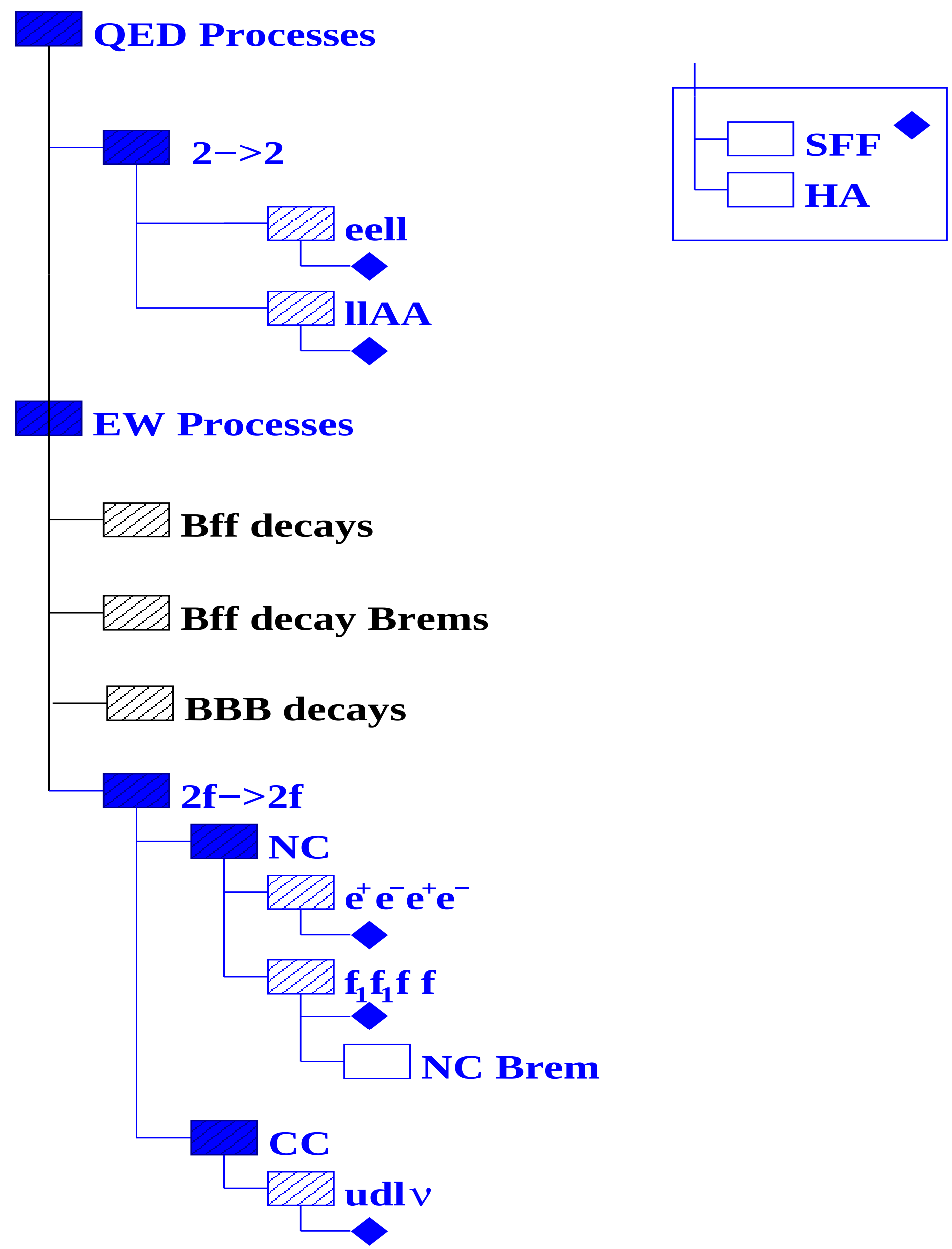
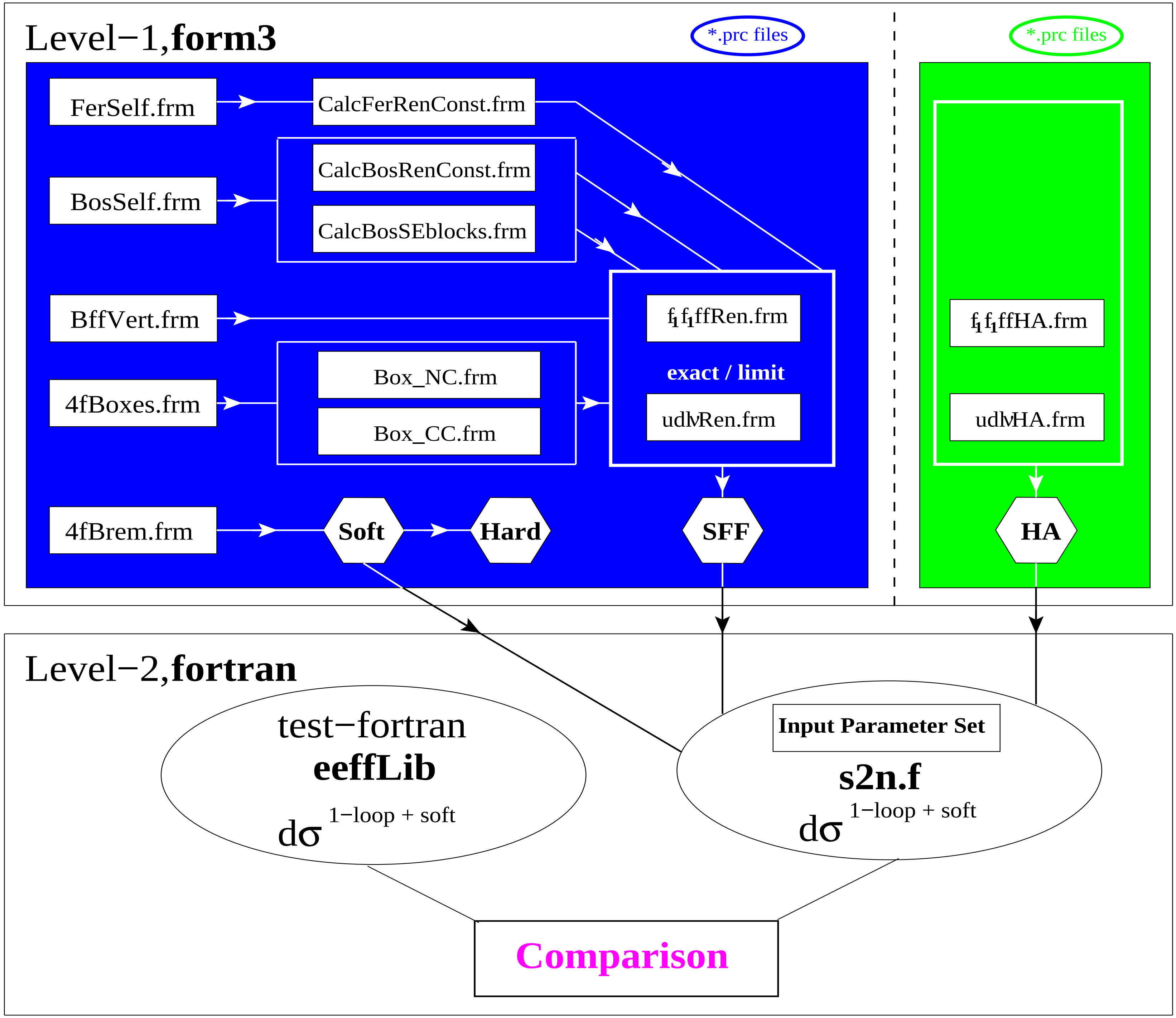
EW part SANC v.0.21




QED Processes
2−>2
eell
llAA
EW Processes
Bff decays
Bff decay Brems
BBB decays
2f−>2f
NC
e+e−e+e−
f1f1f f
NC Brem
CC
udlν
SFF
HA



Level−1,**form3**
*.prc files
*.prc files
FerSelf.frm
CalcFerRenConst.frm
BosSelf.frm
CalcBosRenConst.frm
CalcBosSEblocks.frm
BffVert.frm
$f_1 f_1$ffRen.frm
exact / limit
udlνRen.frm
4fBoxes.frm
Box_NC.frm
Box_CC.frm
$f_1 f_1$ffHA.frm
udlνHA.frm
4fBrem.frm
Soft
Hard
SFF
HA
Level−2,**fortran**
test−fortran
eeffLib
$d\sigma^{1-\text{loop + soft}}$
Input Parameter Set
s2n.f
$d\sigma^{1-\text{loop + soft}}$
Comparison

Automatic Calculation for Processes

COMPLETE ONE-LOOP ElectroWeak

SCALAR FORM FACTORS **(SFF)** & **(HA)** HELICITY AMPLITUDES

- **SFF** & **HA for ANY NC:** $f_1\bar{f}_1 \to f\bar{f}$

- **SFF** & **HA for ANY CC :** $f_1\bar{f}_1^{'} \to f\bar{f}^{'}$,
  but in v.0.21: $\bar{u}d \to l\bar{\nu}$

- **SFF for NC** $f\bar{f}2B \to 0$ **:** $f\bar{f}\gamma\gamma$, $f\bar{f}\gamma Z$, $f\bar{f}\gamma H$

SFF & HA for ANY NC $f_1 \bar{f}_1 \to f \bar{f}$

- R_ξ **gauge**

- f_1 — ν, e, u, d

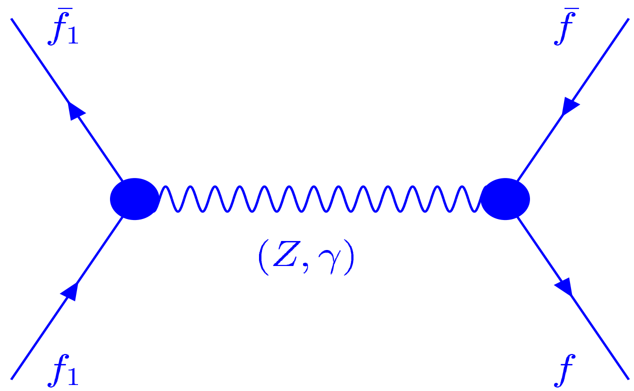
  f — **ANY MASSIVE FERMION**

MASSIVE CASE AND TWO LIMITS

$$\boxed{m_f \longrightarrow 0}$$
$$\boxed{m_{f'} \longrightarrow 0}$$

ref.: A. Andonov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya and G. Nanava, "Update of one-loop corrections for $e^+e^- \to f\bar{f}$, first run of SANC system", 34 N5 *Particles and Nuclei* **(2003) 577-618**

Amplitudes in L, Q, D - basis



ignore electron masses

$$\mathbf{A} \sim$$

$$\left[\mathbf{i}\gamma_\mu\left(\mathbf{1}+\gamma_\mathbf{5}\right)\mathbf{F}_\mathbf{L}^{f_\mathbf{1}}\left(\mathbf{s}\right)+\mathbf{i}\gamma_\mu\mathbf{F}_\mathbf{Q}^{f_\mathbf{1}}\left(\mathbf{s}\right)\right]\otimes$$

$$\left[\mathbf{i}\gamma_\mu\left(\mathbf{1}+\gamma_\mathbf{5}\right)\mathbf{F}_\mathbf{L}^{f}\left(\mathbf{s}\right)+\mathbf{i}\gamma_\mu\mathbf{F}_\mathbf{Q}^{f}\left(\mathbf{s}\right)+\mathbf{m}_f\mathbf{I}\mathbf{D}_\mu\mathbf{F}_\mathbf{D}^{f}\left(\mathbf{s}\right)\right]$$

$$\mathbf{D}_\mu=\left(\mathbf{p_3}-\mathbf{p_4}\right)_\mu$$

Born-like structure

of the ONE LOOP AMPLITUDE in terms of

LL, QL, LQ, QQ, LD and QD form factors

$$
\begin{aligned}
A_\gamma &= i\frac{4\pi Q_{f_1} Q_f}{s}\chi_\gamma(s)\alpha(s)\gamma_\mu \otimes \gamma_\mu \\
\mathcal{A}_Z &= i\frac{g^2}{16\pi^2}\, e^2\, 4 I^{(3)}_{f_1} I^{(3)}_f \frac{\chi_Z(s)}{s} \\
&\times \Big\{ \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5)\, \mathbf{SFF}_{LL}(s,t) \;-4|Q_{f_1}|s^2_W \gamma_\mu \otimes \gamma_\mu(1+\gamma_5)\, \mathbf{SFF}_{QL}(s \\
&- 4|Q_f|s^2_W \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu\, \mathbf{SFF}_{LQ}(s,t) + 16|Q_{f_1} Q_f| s^4_W \gamma_\mu \otimes \gamma_\mu\, \mathbf{SFF}_{QQ}(s,t) \\
&+ \gamma_\mu(1+\gamma_5) \otimes (-i m_f D_\mu)\; \mathbf{SFF}_{LD}(s,t) - 4|Q_{f_1}|s^2_W \gamma_\mu \otimes (-i m_f D_\mu)\; \mathbf{SFF}_{QD}(s
\end{aligned}
$$

$f_1\bar{f}_1 \to f\bar{f}$ process in the Helicity Amplitudes

16 HA for any $2f \to 2f$ process.
For the NC process and if the initial mass is ignored $\longrightarrow$ 6 HA, which depend on kinematical variables and our
6 scalar form factors:

$$
\begin{aligned}
\underline{\underline{\mathbf{HA}_{++++}}} &= 0, \qquad \underline{\underline{\mathbf{HA}_{+++-}}} = 0, \qquad \underline{\underline{\mathbf{HA}_{++-+}}} = 0, \qquad \underline{\underline{\mathbf{HA}_{++--}}} = 0,\\
\underline{\underline{\mathbf{HA}_{+-+-}}} &= s\,(1-\cos\vartheta)\left(Q_{f_1}Q_f\alpha(s) + \chi_Z\delta_{f_1}[\,(1+\beta_f)\,I_f^{(3)}\mathbf{SFF}_{QL} + \delta_f\mathbf{SFF}_{QQ}]\right),\\
\underline{\underline{\mathbf{HA}_{+--+}}} &= s\,(1+\cos\vartheta)\left(Q_{f_1}Q_f\alpha(s) + \chi_Z\delta_{f_1}[(1-\beta_f)I_f^{(3)}\mathbf{SFF}_{QL} + \delta_f\mathbf{SFF}_{QQ}]\right),\\
\underline{\underline{\mathbf{HA}_{+---}}} &= \underline{\underline{\mathbf{HA}_{+-++}}} = 2\sqrt{s}m_f\sin\vartheta\left(Q_{f_1}Q_f\alpha(s) + \chi_Z\delta_{f_1}[I_f^{(3)}\mathbf{SFF}_{QL} + \delta_f\mathbf{SFF}_{QQ} + \frac{1}{2}s{\beta_f}^2 I_f^{(3)}\mathbf{SFF}_{QD}]\right),\\
\underline{\underline{\mathbf{HA}_{-+++}}} &= \underline{\underline{\mathbf{HA}_{-+--}}} = -2\sqrt{s}m_f\sin\vartheta\left(Q_{f_1}Q_f\alpha(s) + \chi_Z\left[2I_{f_1}^{(3)}I_f^{(3)}\mathbf{SFF}_{LL} + 2I_{f_1}^{(3)}\delta_f\mathbf{SFF}_{LQ}\right.\right.\\
&\left.\left. +\delta_{f_1}I_f^{(3)}\mathbf{SFF}_{QL} + \delta_{f_1}\delta_f\mathbf{SFF}_{QQ} + \frac{1}{2}s{\beta_f}^2 I_f^{(3)}(2I_{f_1}^{(3)}\mathbf{SFF}_{LD} + \delta_{f_1}\mathbf{SFF}_{QD})\right]\right),\\
\underline{\underline{\mathbf{HA}_{-++-}}} &= s\,(1+\cos\vartheta)\left(Q_{f_1}Q_f\alpha(s) + \chi_Z\left[\,(1+\beta_f)\,(2I_{f_1}^{(3)}I_f^{(3)}\mathbf{SFF}_{LL} + \delta_{f_1}I_f^{(3)}\mathbf{SFF}_{QL})\right.\right.\\
&\left.\left. +\delta_f\left(2I_{f_1}^{(3)}\mathbf{SFF}_{LQ} + \delta_{f_1}\mathbf{SFF}_{QQ}\right)\,\right]\right),\\
\underline{\underline{\mathbf{HA}_{-+-+}}} &= s\,(1-\cos\vartheta)\left(Q_{f_1}Q_f\alpha(s) + \chi_Z[\,(1-\beta_f)\,I_f^{(3)}(2I_{f_1}^{(3)}\mathbf{SFF}_{LL} + \delta_{f_1}\mathbf{SFF}_{QL}) + \delta_f(2I_{f_1}^{(3)}\mathbf{SFF}_{LQ} + \delta_{f_1}\mathbf{SFF}_{QQ}\right.\\
\underline{\underline{\mathbf{HA}_{--++}}} &= 0, \qquad \underline{\underline{\mathbf{HA}_{--+-}}} = 0, \qquad \underline{\underline{\mathbf{HA}_{---+}}} = 0, \qquad \underline{\underline{\mathbf{HA}_{----}}} = 0.
\end{aligned}
$$

$$
\cos\vartheta = \left(t - m_f^2 + \frac{s}{2}\right)\frac{2}{s\beta_f}, \quad \beta_f^2 = 1 - 4\frac{m_f^2}{s}, \quad \delta_f = v_f - a_f\,.
$$

Differential cross section

for the amplitude $\mathbf{HA}_{\lambda_i\lambda_j\lambda_k\lambda_l}$ each index $\lambda_{(i,j,k,l)}$ takes two values $(\pm = \pm 1)$ meaning 2 times the projection of spins $f_1, \bar{f}_1, f, \bar{f}$ onto their corresponding momentum.

For the unpolarized case:

$$\frac{d\sigma}{d\cos\vartheta} = \frac{\pi\alpha^2}{s^3}\beta_f N_c \sum_{\lambda_i\lambda_j\lambda_k\lambda_l} \left|\mathbf{HA}_{\lambda_i\lambda_j\lambda_k\lambda_l}\right|^2.$$

Comparison $\longrightarrow$ Agreement

$f_1\bar{f}_1 \to f\bar{f}$, **ALL CHANNELS**

- `s2n.f–eeffLib–ZFITTER`

SFFs $\longrightarrow$ **8–9 digits**

Complete 1-loop differential cross section $d\sigma^{(1)}/d\cos\vartheta$
$\longrightarrow$ **7–8 digits**

Total 1-loop cross section and σ^{FB} for the light fermion masses
$\longrightarrow$ **6–7 digits**

Comparison $\longrightarrow$ Agreement

Drell-Yan type processes

$$u\bar{u} \longrightarrow e\bar{e}$$
$$u\bar{u} \longrightarrow \mu\bar{\mu}$$
$$u\bar{u} \longrightarrow \tau\bar{\tau}$$
$$d\bar{d} \longrightarrow e\bar{e}$$
$$d\bar{d} \longrightarrow \mu\bar{\mu}$$
$$d\bar{d} \longrightarrow \tau\bar{\tau}$$

- `s2n.f–eeffLib`

Complete 1-loop differential cross section $d\sigma^{(1)}/d\cos\vartheta$

α and GF scheme

$\longrightarrow$ **9($e\bar{e}$)–14 digits**

Proccess $u + \bar{u} \longrightarrow e + \bar{e}$

α scheme

cost/$\sqrt{s}$	500.0	1000	
-0.900	0.03581972580387	0.00899670297493	
	s2n		
	0.03581972581963	0.00899670297604	effLib
height-0.500	0.04446013767097	0.01071308354911	
	0.04446013767140	0.01071308354910	
0.000	0.09778676798511	0.02246673648080	
	0.09778676798584	0.02246673648085	
0.500	0.19087729262934	0.04269498400766	
	0.19087729262959	0.04269498400772	
0.900	0.28031863553893	0.06096810618306	
	0.28031863548505	0.06096810617686	

GF-scheme

cost/$\sqrt{s}$	500.0	1000
-0.900	0.03547857982171	0.00890403807530
	0.03547857983735	0.00890403807640
-0.500	0.04443025506435	0.01068447174734
	0.04443025506477	0.01068447174733
0.000	0.09818115675064	0.02248135876431
	0.09818115675138	0.02248135876435
0.500	0.19145389752211	0.04262832096331
	0.19145389752233	0.04262832096338
0.900	0.28026130655663	0.06056928257284
	0.28026130650034	0.06056928256636

Proccess $d + \bar{d} \longrightarrow e + \bar{e}$

α scheme

$\cos t/\sqrt{s}$	500.0	1000.0	
-0.900	0.01511430405014	0.00341380884032	s2n
	0.01511430405059	0.00341380883937	ceffLib
-0.500	0.02054686266316	0.00455766213393	
	0.02054686266347	0.00455766213398	
0.000	0.05127896997667	0.01144306437773	
	0.05127896997711	0.01144306437775	
0.500	0.11322875003706	0.02592450527077	
	0.11322875003788	0.02592450527078	
0.900	0.19668817834209	0.04724845288244	
	0.19668817835267	0.04724845288320	

GF-scheme

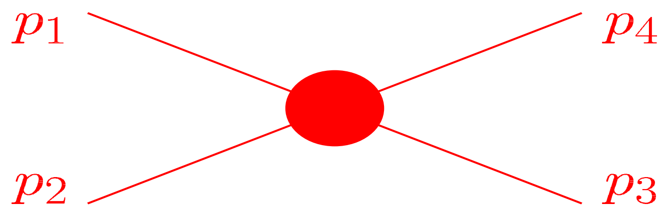
$\cos t/\sqrt{s}$	500.0	1000.0
-0.900	0.01537277741159	0.00346067289523
	0.01537277741220	0.00346067289423
-0.500	0.02076622609081	0.00458558222594
	0.02076622609113	0.00458558222599
0.000	0.05166875336279	0.01148059111328
	0.05166875336324	0.01148059111331
0.500	0.11428327380515	0.02609108409765
	0.11428327380598	0.02609108409766
0.900	0.19924503971416	0.04783376929622
	0.19924503972523	0.04783376929702

SFF & HA for (CC): $ud \to l\nu$

APPLICATIONS:

- **Decay:** $\mathbf{t} \longrightarrow \mathbf{bl}\nu$
- **DIS:** $\bar{\nu}_\mu \mathbf{u} \longrightarrow \mathbf{e}^+\mathbf{d}$ $\longrightarrow$ **talk by A. Arbuzov**
- **Drell-Yan type processes:** $\mathbf{u\bar{d}} \longrightarrow \mathbf{l}^+\nu$

Born-like structure



$$Q^2 = (p_1 + p_2)^2 = -s$$
$$T^2 = (p_2 + p_3)^2 = -t$$
$$U^2 = (p_2 + p_4)^2 = -u$$
$$s + t + u = m_u^2$$

$$\mathcal{A}_{ud \longrightarrow l\nu} \sim \frac{ig^2}{8(M_W^2 + Q^2)} \Big[\quad \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5)\, \mathbf{SFF}_{LL}(s,t,u)$$
$$+ \quad \gamma_\mu(1+\gamma_5) \otimes (-im_u\, \mathcal{D}_\mu)\, \mathbf{SFF}_{LD}(s,t,u) \Big]$$

$$m_u \neq 0$$
$$\mathcal{D}_\mu = p_3 - p_4$$

see ref: D. Bardin, P. Christova, L. Kalinovskaya, '`SANC` Status Report'
Nucl. Phys. B (Proc. Suppl.) **116** (2003) 48–52.

Comparison $\longrightarrow$ Agreement

eeffLib–s2n—ZFITTER

Input parameter: Z-mass = 91.1867, W-mass = 80.4514958, H-mass = 120.

For the **SFF**s **AGREEMENT** within **8-9** digits

s,t,u= 100. -1. -99.	SFF^{QED}_{LL}	(16.607403, 6.41190095)	eeffLib
		16.607403, 6.41190095	s2n
	SFF^{EW}_{LL}	(12.5405065, 0.0155681267)	eeffLib
		12.5405065, 0.0155681267	s2n
	SFF_{LL}	(1.04796481, 0.0168435724)	eeff
		1.04794194	instability in ZFITTER
s,t,u= 100. -50. -50.	SFF^{QED}_{LL}	(45.962368, 6.41190095)	eeffLib
		45.962368, 6.41190095	s2n
	SFF^{EW}_{LL}	(-15.2896247, 0.0157932753)	eeffLib
		-15.2896247, 0.0157932753	s2n
	SFF_{LL}	(1.05196073, 0.0168441624)	eeffLib
		1.05196079	ZFITTER
s,t,u= 100. -99. -1.	SFF^{QED}_{LL}	(113.240851, 6.41190095)	eeffLib
		113.240829, 6.41190095	s2n
	SFF^{EW}_{LL}	(-80.6418362, 0.0160183836)	eeffLib
		-80.6418362, 0.0160183836	s2n
	SFF_{LL}	(1.05700864,0.0168447523)	eeffLib
		1.05700876	ZFITTER

$$\mathbf{SFF}_{\rm LL}(\mathbf{s},\mathbf{t},\mathbf{u}) = \mathbf{1} + \frac{\alpha}{4\pi \mathbf{s}^2_{\rm W}}\mathbf{SFF}^{\rm QED}_{\rm LL}(\mathbf{s},\mathbf{t},\mathbf{u}) + \frac{\alpha}{4\pi \mathbf{s}^2_{\rm W}}\mathbf{SFF}^{\rm EW}_{\rm LL}(\mathbf{s},\mathbf{t},\mathbf{u}) - \mathbf{\Delta r}$$

Conclusion

Status of $2f2\bar{f} \longrightarrow 0$ in SANC

- $$\frac{d\sigma}{d\cos\vartheta} \sim \sum_{\lambda_i\lambda_j\lambda_k\lambda_l} \left| \mathbf{HA}(\mathbf{SFF}^{\mathbf{Born+1-loop+soft}})_{\lambda_i\lambda_j\lambda_k\lambda_l} \right|^2$$

- **hard photons from a partner MC**

SFF for (NC) $f\bar{f}2B \rightarrow 0$:

$\mathbf{f\bar{f}}\gamma\gamma \longrightarrow 0$:

$$
\begin{array}{ccc}
e^+e^- & \longrightarrow & \gamma\gamma \\
q\bar{q} & \longrightarrow & \gamma\gamma \\
\gamma\gamma & \longrightarrow & t\bar{t} \\
\gamma e & \longrightarrow & \gamma e
\end{array}
$$

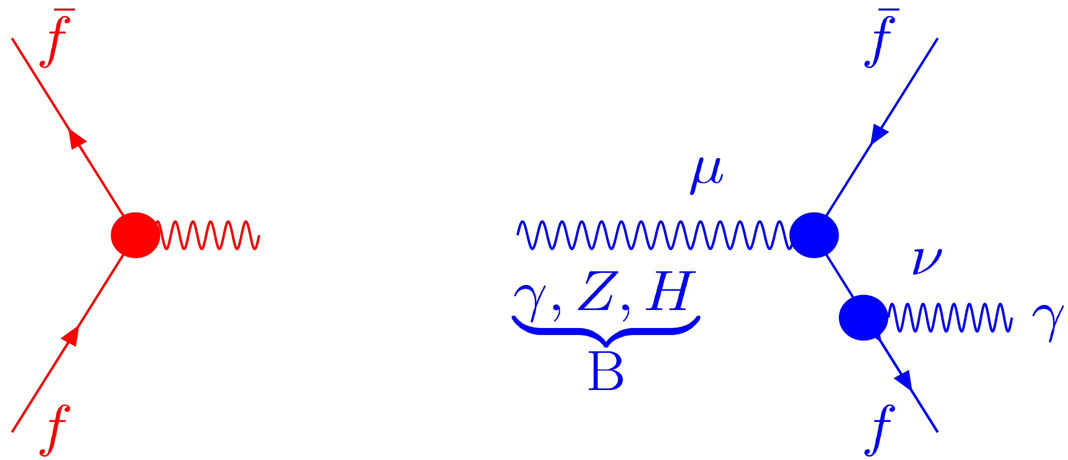
$\mathbf{f\bar{f}Z}\gamma \longrightarrow 0$:

$$
\begin{array}{ccc}
Z & \longrightarrow & f\bar{f}\gamma \\
e^+e^- & \longrightarrow & Z\gamma \\
q\bar{q} & \longrightarrow & Z\gamma \\
\gamma e & \longrightarrow & Ze
\end{array}
$$

$\mathbf{f\bar{f}H}\gamma \longrightarrow 0$:

$$
\begin{array}{ccc}
H & \longrightarrow & f\bar{f}\gamma \\
e^+e^- & \longrightarrow & H\gamma \\
q\bar{q} & \longrightarrow & H\gamma \\
\gamma e & \longrightarrow & He
\end{array}
$$

Amplitudes $B \to f\bar{f}\gamma$ in chosen basis

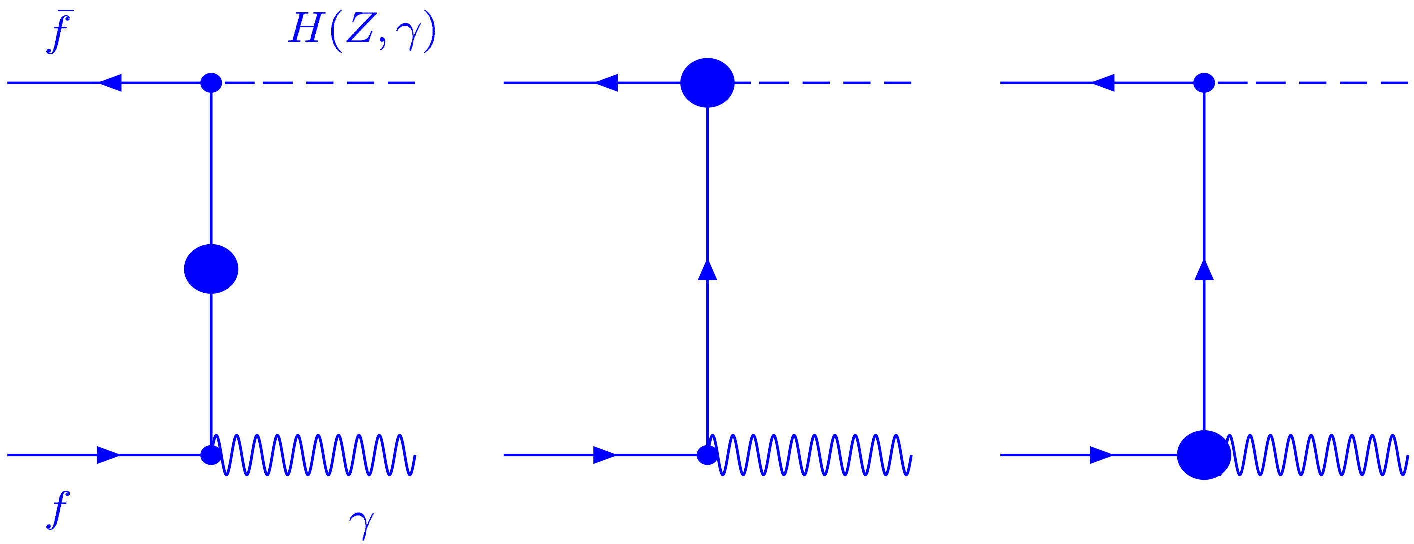


$$A_{\mu\nu} = \sum_{\mathrm{i}=1}^{20} \mathbf{SFF}_{\mathrm{i}} \cdot \mathrm{Structures}_{\mathrm{i}}(1, \gamma_5)_{\mu\nu}$$

40 40 12 STRUCTURES

$$\underbrace{\gamma \qquad Z \qquad H}_{\mathrm{B}}$$

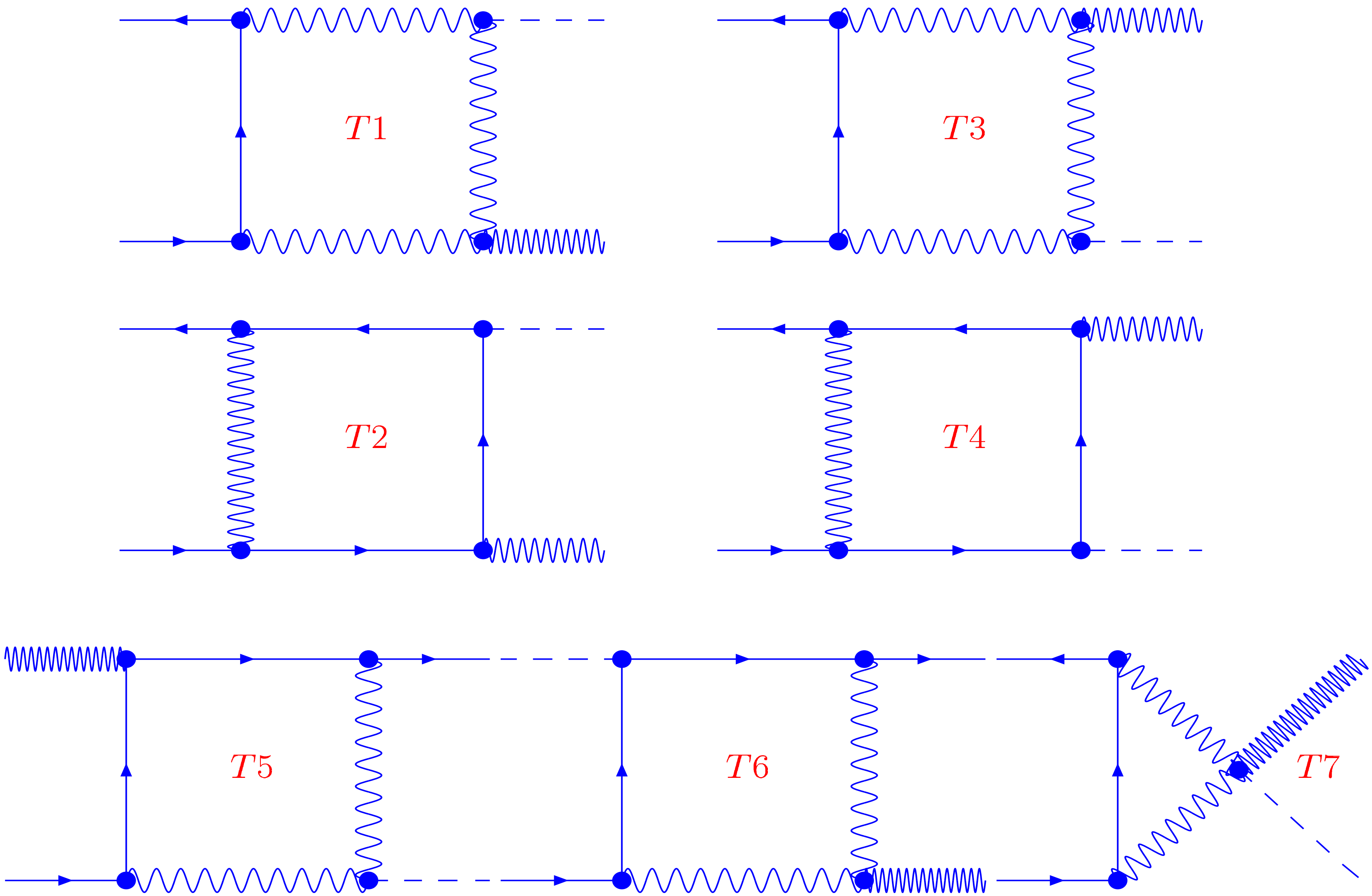
DATABASE ideology of BUILDING BLOCKS



Self-Energies **Vertices**

BANK OF PRECOMPUTED FEYNMAN DIAGRAMS
FULLY MASSIVE CASE !

Boxes topology



Gauge-invariant subsets of diagrams

		k
• $\{A \longrightarrow \xi_A\}$	**A CLUSTER**	1
• $\{Z, \phi^0 \longrightarrow \xi_Z\}$	**Z CLUSTER**	2
• $\{W, \phi^{\pm} \longrightarrow \xi\ \}$	**W CLUSTER**	3
• $\{H, \phi^0 \longrightarrow \xi_Z\}$	**H CLUSTER**	4

Minimum set of structures for $H \longrightarrow f\bar{f}\gamma$

$$\mathbf{A}_\nu = \sum_{\mathbf{k=1}}^{4} \mathbf{A_k} = \sum_{k=1}^{4} \Big(\mathbf{SFF_1^k}\, i\gamma_\nu + \mathbf{SFF_2^k}\, \mathbf{p_{1\nu}} + \mathbf{SFF_3^k}\, \mathbf{p_{2\nu}} + \mathbf{SFF_4^k}\, \hat{\mathbf{p}}_\gamma \gamma_\nu + \mathbf{SFF_5^k}\, i\hat{\mathbf{p}}_\gamma \mathbf{p_{1\nu}} + \mathbf{SFF_6^k}\, i\hat{\mathbf{p}}_\gamma \mathbf{p_{2\nu}} \Big) \big(\mathbf{1}; \gamma_\mathbf{5}\big)$$

$$\mathbf{A}_\nu = \sum_{\mathbf{k=1}}^{4} \mathbf{A_k} = \sum_{\mathbf{k=1}}^{4} \Big(\mathbf{SFF_1^k} \Big[\frac{\mathbf{1}}{\mathbf{u - m_f^2}} \mathbf{p_{1\nu}} - \frac{\mathbf{1}}{\mathbf{t - m_f^2}} \mathbf{p_{2\nu}}\Big] + \mathbf{SFF_2^k}\, \hat{\mathbf{p}}_\gamma \gamma_\nu - \mathbf{SFF_3^k}\, i\Big[\frac{\mathbf{1}}{\mathbf{u - m_f^2}} \mathbf{p_{1\nu}} + \frac{\mathbf{1}}{\mathbf{2}} \gamma_\nu\Big] - \mathbf{SFF_4^k}\, i\Big[\frac{\mathbf{1}}{\mathbf{t - m_f^2}} \mathbf{p_{2\nu}} + \frac{\mathbf{1}}{\mathbf{2}} \gamma_\nu\Big] \Big) \big(\mathbf{1}; \gamma_\mathbf{5}\big)$$

Limit case $H \to d\bar{d}\gamma$

$$\mathrm{SFFV}_3^2 = s_W \left(-\frac{5}{4c_W^3} + \frac{1}{c_W} - 2c_W\right) \frac{M_Z}{27} F_2^{lim}(-u,-t)$$

$$\mathrm{SFFV}_4^2 = s_W \left(-\frac{5}{4c_W^3} + \frac{1}{c_W} - 2c_W\right) \frac{M_Z}{27} F_2^{lim}(-t,-u)$$

$$\mathrm{SFFA}_3^2 = \frac{s_W}{9c_W} \left(\frac{1}{4c_W^2} - 1\right) M_Z F_2^{lim}(-u,-t)$$

$$\mathrm{SFFA}_4^2 = \frac{s_W}{9c_W} \left(\frac{1}{4c_W^2} - 1\right) M_Z F_2^{lim}(-t,-u)$$

$$\begin{aligned} F_2^{lim}(-t,-u) = & \left(M_Z^2 - t\right) \left[\frac{(M_Z^2 - t)}{st}(M_H^2 - s) + \frac{t}{s}\right] D_0(0,-m_d^2,-M_H^2,-m_d^2,-t,-u,m_d,m_d,M \\ & + \frac{(M_Z^2 - t)}{st}[(M_H^2 - t)\; C_0(\,-m_d^2,-M_H^2,-t;m_d,M_Z,M_Z) \\ & - t\; C_0(0,-m_d^2,-t;m_d,m_d,M_Z) - u\; C_0(0,-m_d^2,-u;m_d,m_d,M_Z)\,] \\ & + \left[\left(M_Z^2 - t\right)\frac{(t-s)}{st} - \frac{2}{u - M_H^2} M_Z^2\right] C_0(\,-m_d^2,-M_H^2,-u;m_d,M_Z,M_Z) \\ & + 2\frac{1}{u - M_H^2}\left[B_0^F(\,-u;M_Z,0) - B_0^F(\,-M_H^2;M_Z,M_Z)\right] \end{aligned}$$

Checking

- ξ **:** ξ_A, ξ_Z, ξ

- **cancellation of UV poles**

- **vanishing of EWFF in front of CP-odd structures,**

$\gamma\gamma \to t\bar{t}$**: 40 Structures** $\longmapsto$ **24**

- **Ward identities:** $A_{\mu\nu} \cdot (p_\gamma)_\nu = 0$, $A_{\mu\nu} \cdot (p_\gamma)_\mu = 0$,

transverse nature of photons: 24 Structures $\longmapsto$ **13**