ECFA-DESY Workshop, Saint Malo, 13 April, 2002

Update of one-loop corrections for $e^+e^- \to f\bar{f}$, first run of `CalcPHEP` system

L. V. Kalinovskaya

Laboratory of Nuclear Physics,

Joint Institute for Nuclear Research, Dubna, Russia

based on:

hep-ph/0202112

hep-ph/0012080

ACKNOWLEDGEMENTS

We would like to thank W. Hollik and C. Schappacher for a discussion of issues of the comparison with FeynArts.

We acknowledge a common work on numerical comparison with J. Fleischer, A. Leike, T. Riemann, and A. Werthenbach.

PURPOSES

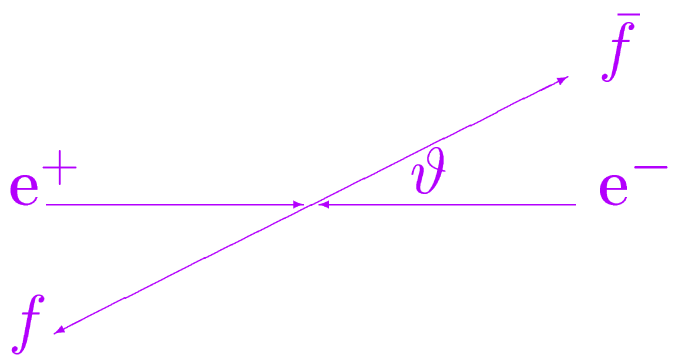
- to create a new EW library of analytical results for the one-loop amplitude of $eeff$ process in the spirit of the book 'The Standard Model in the Making', where the process $e^+e^- \to t\bar{t}$ was not covered.
- main goal of this calculation – to create a platform for the study of other processes within `CalcPHEP`.
- to cross-check the `CalcPHEP` results against the results of the other existing codes for a rather complicated $2 \to 2$ process at one-loop level.
- to demonstrate gauge invariance in R_ξ by examining cancellation of gauge parameters and to search for gauge-invariant subsets of diagrams;
- to offer a possibility to compare the results with those in the unitary gauge, as a cross-check;
- to write a `FORTRAN` code, `eeffLib.f` for an internal cross-check of another code, which was created automatically using the `s2n.f` software — a part of `CalcPHEP` system, thereby benchmarking `s2n.f` software.

THE ONE LOOP AMPLITUDE

$$e^+e^- \longrightarrow f\bar{f}$$

- TWO GAUGES $\longrightarrow R_\xi$ AND THE UNITARY

- AUTOMATIC CALCULATION BY CalcPHEP

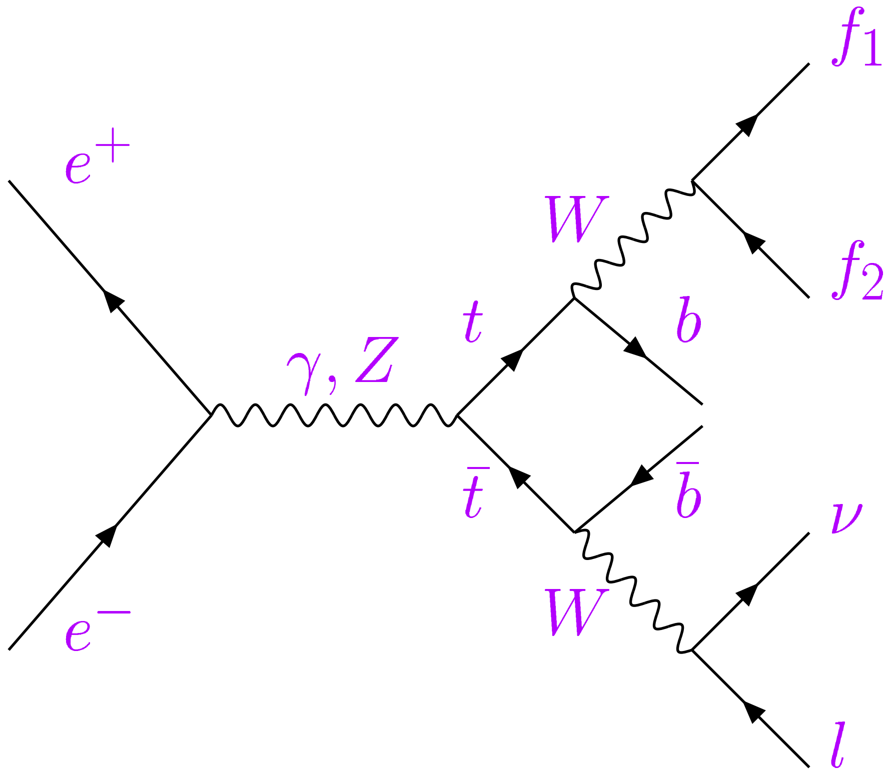


$$\left(\frac{d\sigma^{\text{virt+soft}}}{d\cos\vartheta}\right)^{eeff}$$

P.S. OMS RENORMALIZATION SCHEME

a systematic presentation – the book

D.Bardin, G.Passarino *The Standard Model in the Making*, **Oxford University Press, Oxford, 1999.**



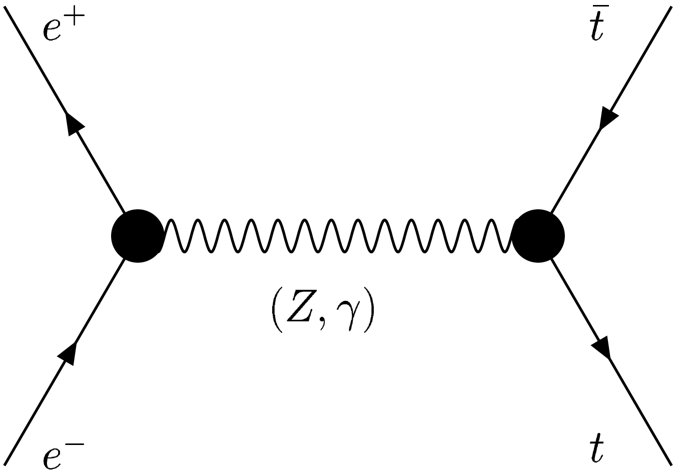
e^+
e^-
γ, Z
t
$\bar{t}$
W
W
b
$\bar{b}$
f_1
f_2
ν
l

AMPLITUDES in L, Q, D - BASIS

MAIN REASON FOR LQD BASIS – CONVENIENCE FOR

THE STUDY OF CANCELLATION OF ξ – DEPENDENT

TERMS



ignore electron masses

$$A \sim$$

$$\left[i\gamma_\mu \left(1+\gamma_5\right) F_L^e\left(s\right) + i\gamma_\mu F_Q^e\left(s\right)\right] \otimes$$

$$\left[i\gamma_\mu \left(1+\gamma_5\right) F_L^t\left(s\right) + i\gamma_\mu F_Q^t\left(s\right) + m_t I D_\mu F_D^t\left(s\right)\right]$$

$$D_\mu = \left(p_3 - p_4\right)_\mu$$

Born-like structure of the ONE LOOP AMPLITUDE in terms of LL, QL, LQ, QQ, LD and QD form factors

$$A_\gamma = i\frac{4\pi Q_e Q_f}{s}\chi_\gamma(s)\alpha(s)\gamma_\mu \otimes \gamma_\mu$$

$$\begin{aligned}
\mathcal{A}_Z = & \; i\frac{g^2}{16\pi^2}e^2\,4I_e^{(3)}I_t^{(3)}\frac{\chi_Z(s)}{s} \\
& \times\Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5)F_{LL}(s,t) \\
& -4|Q_e|s_W^2\gamma_\mu\otimes\gamma_\mu(1+\gamma_5)F_{QL}(s,t) \\
& -4|Q_t|s_W^2\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu F_{LQ}(s,t) \\
& +16|Q_e Q_t|s_W^4\gamma_\mu\otimes\gamma_\mu F_{QQ}(s,t) \\
& +\gamma_\mu(1+\gamma_5)\otimes(-im_t D_\mu)\,F_{LD}(s,t) \\
& -4|Q_e|s_W^2\gamma_\mu\otimes(-im_t D_\mu)\,F_{QD}(s,t)\Big\}
\end{aligned}$$

Every form factor in R_ξ gauge could be represented as a sum

$$F^{\xi}_{L,Q,D}(s) = F^{(1)}_{L,Q,D}(s) + F^{add}_{L,Q,D}(s,\xi)$$

First term corresponds to $\xi = 1$ gauge and the second contains all ξ dependences and vanishes for $\xi = 1$ by construction.

CHECKING THE CANCELLATION

of the additional terms we found seven gauge-invariant subsets of diagrams

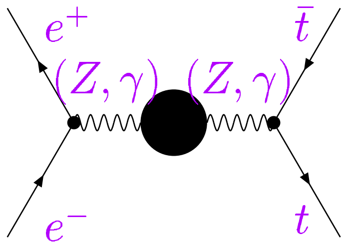
- $\left\{A \longrightarrow \xi_A\right\}$ – **VERTICES + WF REN A CLUSTER**
- $\left\{A \longrightarrow \xi_A\right\}$ – **AA BOX**
- $\left\{A, Z \longrightarrow \xi_A \xi_Z\right\}$ – **ZA BOX**
- $\left\{Z \longrightarrow \xi_Z\right\}$ – **ZZ BOX**
- $\left\{Z, \phi^0 \longrightarrow \xi_Z\right\}$ **Z CLUSTER**
- $\left\{H, \phi^0 \longrightarrow \xi_Z\right\}$ **H CLUSTER**
- $\left\{W, \phi^{\pm} \longrightarrow \xi\right\}$ **W CLUSTER**

+ **self-energies and WW box**

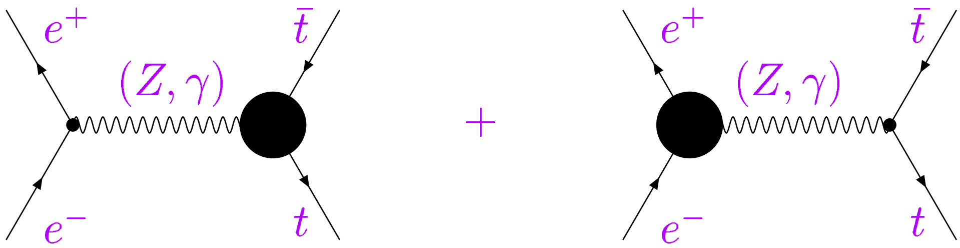
BUILDING BLOCKS

in the order of increasing complexity:

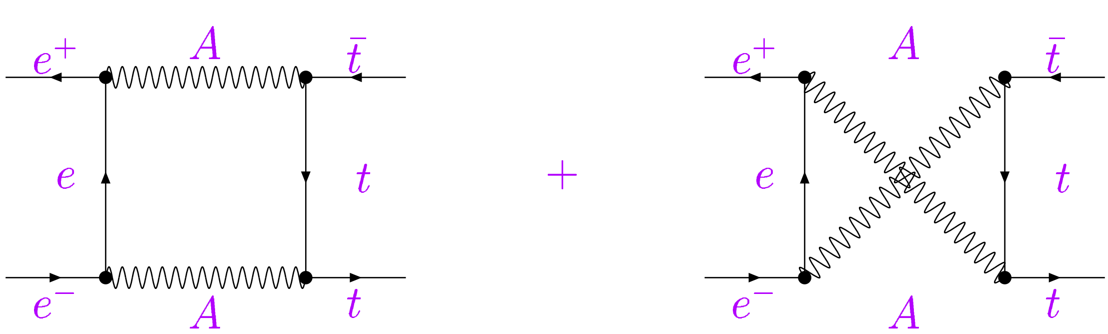
SELF-ENERGIES



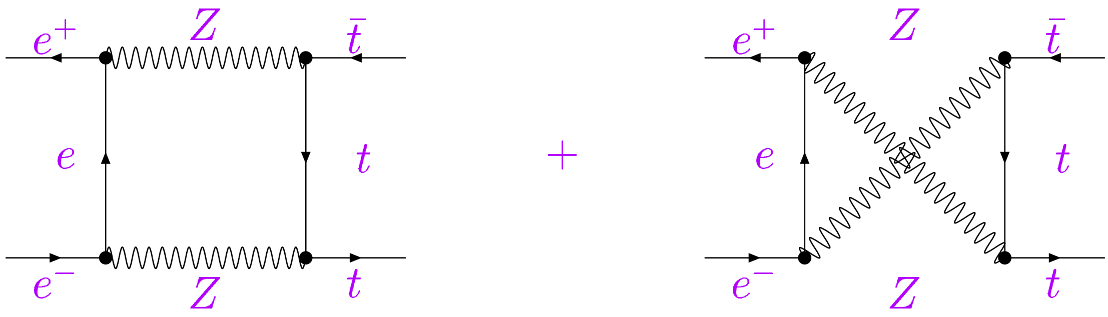
VERTICES



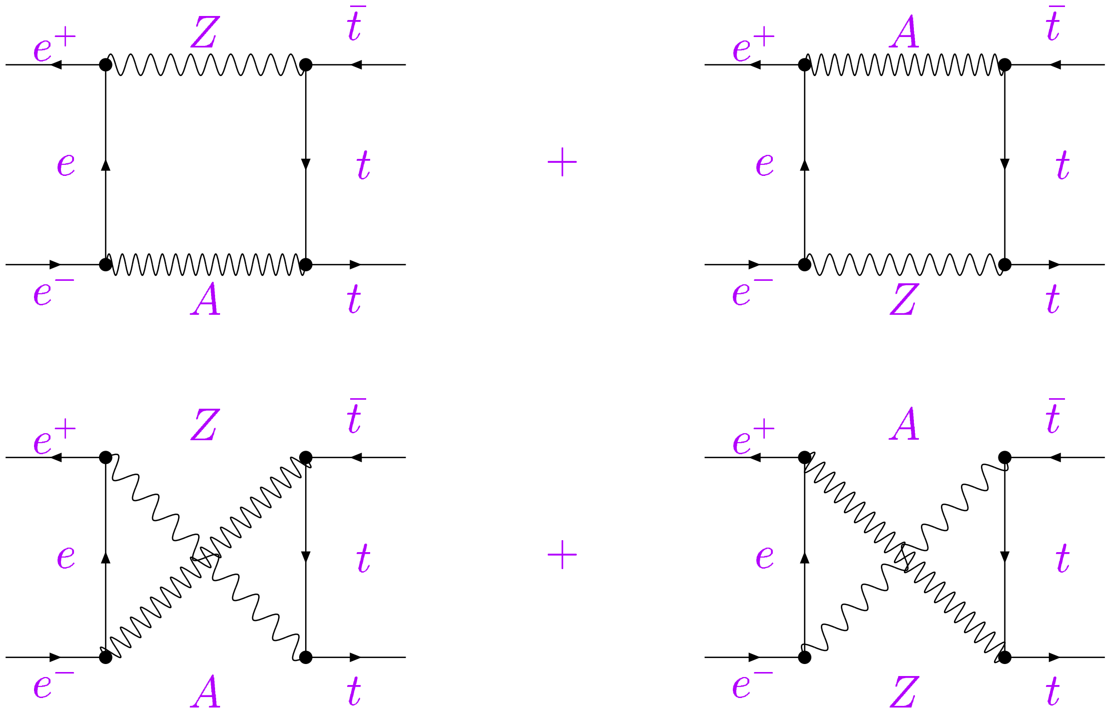
BOXES



Direct and crossed AA boxes.



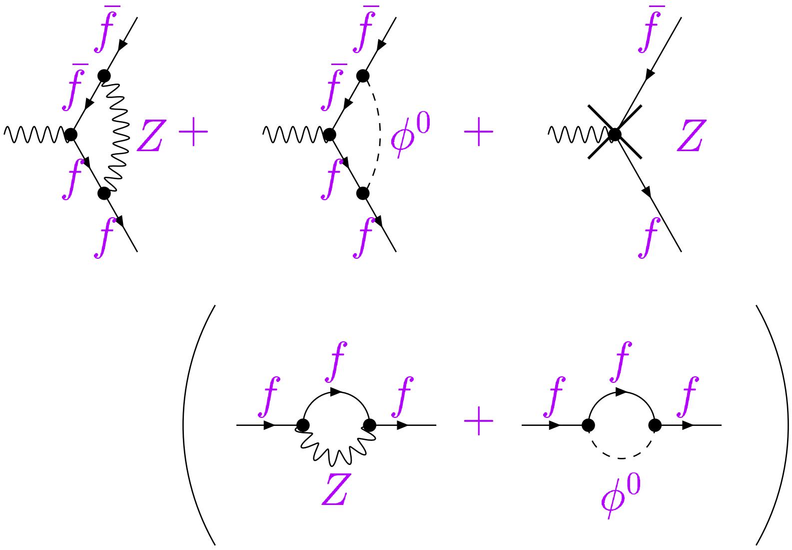
Direct and crossed ZZ boxes.



Direct and crossed ZA boxes.

ϕ^0 **and** $\phi^\pm$ **do not contribute since we ignore** m_e

FORM FACTORS of Z CLUSTER



Separating out pole contributions $1/\bar{\epsilon}$, we define finite quantities. We note, that if a form factor $F_A^{ij}(s)$ has a pole, than the corresponding finite part $\mathcal{F}_A^{ij}(s)$ is μ-dependent.

$$F_L^{\gamma Z}(s) = \mathcal{F}_L^{\gamma Z}(s)$$

$$F_Q^{\gamma Z}(s) = \mathcal{F}_Q^{\gamma Z}(s)$$

$$F_D^{\gamma Z}(s) = \mathcal{F}_D^{\gamma Z}(s)$$

$$F_L^{zZ}(s) = -\frac{1}{4} r_{tW} \frac{1}{\bar{\varepsilon}} + \mathcal{F}_L^{zZ}(s)$$

$$F_Q^{zZ}(s) = -\frac{1}{16} \frac{1}{|Q_t|\, s_W^2} r_{tW} \frac{1}{\bar{\varepsilon}} + \mathcal{F}_Q^{zZ}(s)$$

$$F_D^{zZ}(s) = \mathcal{F}_D^{zZ}(s)$$

$e^+e^- \to t\bar{t}$ process in the helicity amplitudes

16 helicity amplitudes for any $2f \to 2f$ process. The unpolarized case, the electron mass is ignored $\longrightarrow$ 6 helicity amplitudes, which depend on kinematical variables, coupling constants and our 6 scalar form factors:

$$
\begin{aligned}
\mathcal{A}_{++++} &= 0, \qquad \mathcal{A}_{+++-} = 0, \qquad \mathcal{A}_{++-+} = 0, \qquad \mathcal{A}_{++--} = 0,\\
\mathcal{A}_{+-+-} &= s\,(1-\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\delta_e\Big[(1+\beta_t)\,I_t^{(3)}F_{QL} + \delta_tF_{QQ}\Big]\right),\\
\mathcal{A}_{+--+} &= s\,(1+\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\delta_e\Big[(1-\beta_t)I_t^{(3)}F_{QL} + \delta_tF_{QQ}\Big]\right),\\
\mathcal{A}_{+---} &= \mathcal{A}_{+-++} = 2\sqrt{s}m_t\sin\vartheta\Bigg(Q_eQ_tF_{GG}\\
&\quad +\chi_Z\delta_e\Big[I_t^{(3)}F_{QL} + \delta_tF_{QQ} + \frac{1}{2}s{\beta_t}^2I_t^{(3)}F_{QD}\Big]\Bigg),\\
\mathcal{A}_{-+++} &= \mathcal{A}_{-+--} = -2\sqrt{s}m_t\sin\vartheta\Bigg(Q_eQ_tF_{GG}\\
&\quad +\chi_Z\Big[2I_e^{(3)}I_t^{(3)}F_{LL} + 2I_e^{(3)}\delta_tF_{LQ} + \delta_eI_t^{(3)}F_{QL} + \delta_e\delta_tF_{QQ}\\
&\quad +\frac{1}{2}s{\beta_t}^2I_t^{(3)}\left(2I_e^{(3)}F_{LD} + \delta_eF_{QD}\right)\Big]\Bigg),\\
\mathcal{A}_{-++-} &= s\,(1+\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\Big[(1+\beta_t)\left(2I_e^{(3)}I_t^{(3)}F_{LL} + \delta_eI_t^{(3)}F_{QL}\right)\right.\\
&\quad \left.+\delta_t\left(2I_e^{(3)}F_{LQ} + \delta_eF_{QQ}\right)\Big]\right),\\
\mathcal{A}_{-+-+} &= s\,(1-\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\Big[(1-\beta_t)\,I_t^{(3)}\left(2I_e^{(3)}F_{LL} + \delta_eF_{QL}\right)\right.\\
&\quad \left.+\delta_t\left(2I_e^{(3)}F_{LQ} + \delta_eF_{QQ}\right)\Big]\right),\\
\mathcal{A}_{--++} &= 0, \qquad \mathcal{A}_{--+-} = 0, \qquad \mathcal{A}_{---+} = 0, \qquad \mathcal{A}_{----} = 0.
\end{aligned}
$$

$$
\cos\vartheta = \left(t - m_t^2 + s/2\right)2/(s\beta_t)\,, \quad \beta_t^2 = 1 - 4m_t^2/s, \quad \delta_f = v_f - a_f.
$$

For the amplitude $\mathcal{A}_{\lambda_i\lambda_j\lambda_k\lambda_l}$ each index $\lambda_{(i,j,k,l)}$ takes two values $(\pm = \pm 1)$ meaning 2 times the projection of spins $e^+, e^-, t, \bar{t}$ onto their corresponding momentum.

$$\frac{d\sigma}{d\cos\vartheta} = \frac{\pi\alpha^2}{s^3}\beta_t N_c \sum_{\lambda_i\lambda_j\lambda_k\lambda_l} \left|\mathcal{A}_{\lambda_i\lambda_j\lambda_k\lambda_l}\right|^2$$

The expression contains, however, spurious contributions of the two-loop order (squares of one-loop terms), which should be suppressed, since we would like to have the one-loop result.

A simple trick.

At the one-loop level LL, LQ, QL, QQ form factors may be represented as:

$$\mathbf{F}_{IJ} = 1 + \frac{\alpha}{4\pi s_W^2} F_{IJ}\,, \quad \textbf{for} \quad IJ = LL,\, LQ,\, QL,\, QQ$$

and

$$\mathbf{F}_{IJ} = \frac{\alpha}{4\pi s_W^2} F_{IJ}\,, \quad \textbf{for} \quad IJ = LD,\, QD.$$

Instead, for the four form factors we write:

$$\mathbf{F}_{IJ} = Z + \frac{\alpha}{4\pi s_W^2} F_{IJ}\,,$$

Then the one-loop result is apparently equal:

$$\frac{d\sigma^{(1)}}{d\cos\vartheta} = \frac{d\sigma}{d\cos\vartheta}[Z=1] - \frac{d\sigma}{d\cos\vartheta}[Z=0].$$

Total scalar form factors of the one-loop amplitude

The ultraviolet-finite results for six scalar form factors are:

$$F_{LL}(s,t,u) = \left[\mathcal{F}^{zee}_{L}(s) + \mathcal{F}^{A,e}(s)\right] + \mathcal{F}^{ztt}_{L}(s) + \mathcal{F}^{ct}_{LL}(s) + 16k\mathcal{F}^{\rm BOX}_{LL}(s,t,u),$$

$$F_{QL}(s,t,u) = \left[\mathcal{F}^{zee}_{Q}(s) + \mathcal{F}^{A,e}(s)\right] + \mathcal{F}^{ztt}_{L}(s) + k\ \mathcal{F}^{\gamma tt}_{L}(s) + \mathcal{F}^{ct}_{QL}(s) + 16k\mathcal{F}^{\rm BOX}_{QL}(s,t,u),$$

$$F_{LQ}(s,t,u) = \left[\mathcal{F}^{zee}_{L}(s) + \mathcal{F}^{A,e}(s)\right] + \mathcal{F}^{ztt}_{Q}(s) + k\ \mathcal{F}^{\gamma ee}_{L}(s) + \mathcal{F}^{ct}_{LQ}(s) + 16k\mathcal{F}^{\rm BOX}_{LQ}(s,t,u),$$

$$F_{QQ}(s,t,u) = \left[\mathcal{F}^{zee}_{Q}(s) + \mathcal{F}^{A,e}(s)\right] + \mathcal{F}^{ztt}_{Q}(s) - \frac{k}{s^2_W}\left[\mathcal{F}^{\gamma ee}_{Q}(s) + \mathcal{F}^{A,e}(s) + \mathcal{F}^{\gamma tt}_{Q}(s)\right] + \mathcal{F}^{ct}_{QQ}(s) + 16k\mathcal{F}^{\rm BOX}_{QQ}(s,t,u),$$

$$F_{LD}(s,t,u) = \mathcal{F}^{ztt}_{D}(s) + 16k\mathcal{F}^{\rm BOX}_{LD}(s,t,u),$$

$$F_{QD}(s,t,u) = \mathcal{F}^{ztt}_{D}(s) + k\ \mathcal{F}^{\gamma tt}_{D}(s) + 16k\mathcal{F}^{\rm BOX}_{QD}(s,t,u),$$

$$k = c^2_W\left(M^2_Z/s - 1\right).$$

For the $IJ = LL$ component of a box contribution we have:

$$\mathcal{F}^{\rm BOX}_{IJ}(s,t,u) = k^{AA}\mathcal{F}^{AA}_{IJ}(s,t,u) + k^{ZA}\mathcal{F}^{ZA}_{IJ}(s,t,u) + k^{ZZ}\mathcal{F}^{ZZ}_{IJ}(s,t,u) + k^{WW}\mathcal{F}^{WW}_{IJ}(s,t,u)$$

and for the other components $IJ = LQ, QL, QQ, LD, QD$ of the box form factors the WW box does not contribute. Moreover,

$$\mathcal{F}^{\gamma(z)tt}_{L,Q,D}(s) = \sum_{B=A,Z,H,W} \mathcal{F}^{\gamma(z)B}_{L,Q,D}(s),$$

except for $\mathcal{F}^{\gamma A}_{L}(s) = 0$ and $\mathcal{F}^{\gamma H}_{L}(s) = 0$.

COMPARISON OF eeffLib–ZFITTER

ELECTROWEAK SCALAR FORM FACTORS

$\sqrt{s}$	100 GeV	200 GeV	300 GeV
F_{LL}	$12.89584 - i1.84784$	$8.24736 - i10.64666$	$8.98375 - i12.88478$
F_{LL}^{Zf}	$12.89583 - i1.84786$	$8.24736 - i10.64651$	$8.98370 - i12.88466$
F_{QL}	$29.30447 + i3.67330$	$29.38219 + i2.27610$	$31.59711 + i1.59302$
F_{QL}^{Zf}	$29.30445 + i3.67330$	$29.38216 + i2.27613$	$31.59710 + i1.59304$
F_{LQ}	$29.10829 + i3.26972$	$29.48510 + i0.92307$	$31.65836 - i0.89713$
F_{LQ}^{Zf}	$29.10832 + i3.26973$	$29.48512 + i0.92312$	$31.65835 - i0.89711$
F_{QQ}	$44.88226 + i8.85688$	$43.31855 + i9.48287$	$44.18773 + i10.25200$
F_{QQ}^{Zf}	$44.88228 + i8.85688$	$43.31854 + i9.48286$	$44.18773 + i10.25196$

TOTAL CROSS SECTION in pb

100 GeV		200 GeV		300 GeV	
σ_{tot}	σ_{FB}	σ_{tot}	σ_{FB}	σ_{tot}	σ_{FB}
160.8981	**70.98416**	**5.021810**	**3.360848**	**2.031754**	**1.269556**
160.8980	**70.98406**	**5.021808**	**3.360848**	**2.031754**	**1.269556**
0.0001	**0.00010**	**0.000002**	**0.0**	**0.0**	**0.0**

the first row – σ^L_{tot}**, i.e.** eeffLib ($m_t = 0.1$ **GeV);**

the second row – σ^Z_{tot}**, i.e.** ZFITTER ($u\bar{u}$ **channel);**

the third row shows the absolute deviation $\sigma^L_{\text{tot}} - \sigma^Z_{\text{tot}}$.

COMPARISON WITH A CODE GENERATED BY s2n_f

The table eeffLib–s2n_f for the complete one-loop differential cross-sections $d\sigma^{(1)}/d\cos\vartheta$, for the standard input parameter set.

— AGREEMENT within 12 or 13 digits.

$\sqrt{s}$	400 GeV	700 GeV	1000 GeV
$\cos\vartheta$			
−0.9	0.22357662754774	0.06610825350063	0.02926006442715
	0.22357662754769	0.06610825350063	0.02926006442715
0.0	0.34494634728716	0.14342802645636	0.06752160108814
	0.34494634728707	0.14342802645634	0.06752160108813
0.9	0.54806778978208	0.33837133344667	0.16973989931024
	0.54806778978194	0.33837133344664	0.16973989931023

first row eeffLib

$m_t = 173.8$ **GeV**

second row s2n_f

COMPARISON WITH THE OTHER CODES

- For the 1-loop cross-section without soft

  — AGREEMENT 11 digits with FeynArts

- For the 1-loop cross-section, with soft

  — AGREEMENT within 7 or 8 digits with Bielefeld–Zeuthen (hep-ph/0202102, J. Fleisher et al.)

$\sqrt{s}$	400 GeV	700 GeV	1000 GeV
$\cos\vartheta$			
–0.9	0.17613018248935	0.05199100267864	0.02310170508071
–0.5	0.21014509428358	0.06560630503586	0.02882301902010
0.0	0.27268108572063	0.11496514450150	0.05495088904853
0.5	0.35592722356682	0.19615154401629	0.09941700898317
0.9	0.43637377538440	0.27915043976042	0.14426233253975

$\frac{d\sigma^{(1)}}{d\cos\vartheta}$ for the process $e^+e^- \to t\bar{t}$ with soft photons, $E_\gamma^{\max} = \sqrt{s}/10$.

CONCLUSION

CalcPHEP basement for treatment of

any $2f \to 2f$ process

(FULLY MASSIVE CASE)

is completetly ready