Meeting on Particle Physics Phenomenology

CERN-TH, Friday 20 October 2000

D. Bardin and L. Kalinovskaya

LNP JINR Dubna Russia

12 Years of Precision Calculations for LEP. What's Next?

OUTLINE

1. Short historical overview.
2. Book heritage:
   - the site `brg.jinr.ru` and CalcPHEP project.
3. New calculation for $e^+e^- \to t\bar{t}$:
   - the `topfit` project.
4. "Algebraic" approach to multiloops (F. Tkachov):
   - "lifting" of polynomial powers;
   - an example in one-loop field;
     - *reduction in n-dimensions*;
     - *first numerical experience.*
5. Outlook

LEP great steps:

- 1989: start;
- 1995 – finish of Z resonance:
  - 150 pb^{-1} delivered to each experiment;
  - ~ 17 Z events;
  - unprecedented experimental accuracy $\leq 10^{-3}$.
- $\geq$ 1995 – work above Z resonance:
  - ≥ 500 pb^{-1} delivered to each experiment;
  - even at high energies accuracy $\leq 1\%$.
- LEP running is extended!
- 2/11/2000: The END?

Challenge for theoreticians — to perform calculations with theoretical uncertainty better than experimental errors!

Theory great steps:

- 60's: S. L. Glashow, S. Weinberg and A. Salam — foundation of the SM — NP 1979.
- 60's – 80's: wide community of theorists creates a framework for precision calculations in HEP.
- 90's: dedicated theoretical support of LEP experiments.
- Recognized by the decision to award NP 1999 to G. t'Hooft and M. Veltman for "having placed this theory (SM) on a firmer mathematical foundation".

May we say that in recent years a new discipline has been born: Precision High-Energy Physics, PHEP:

1. *experimental measurements at per mill level;*
2. *supporting theoretical calculations?*

LEP1 and SLAC — bright examples!

TEVATRON approaches PHEP standards.

LHC expects to be a typical PHEP facility.

LC, especially its GigaZ phase.

Me and my colleagues were involved within `ZFITTER` *project*

`ZFITTER` **main steps:**

1. 1976–1982: papers on *on-mass-shell renormalization scheme*;
2. 1982–1991: papers on one-loop EWRC for Z observables;
3. 1989: WS "Z-Physics at LEP" — first `ZFITTER` was born;
4. 1989–1992: papers on QED RC in *flux function language*;
5. 1992: **`ZFITTER v4.5`**, CERN-TH 6443/92;
6. 1994: "Precision calculations for Z resonance", CERN 95-03;
7. 1999: **`ZFITTER v6.21`**, *CPC to appear;*
8. 1999–2000: LEP2 MC WS, 2f group, v.6.30;
9. **1989–2000: `ZFITTER` upgrade and support in ADLO, SLC and LEPEWWG — TWELVE YEARS.**
10. 12/2000: The END of `ZFITTER` project. **What's next?**

- 1997-1998: The Book: *DB* & *Giampiero Passarino* "The Standard Model in the Making", OUP 1999.

LEP tandems

`ZFITTER`

D.Bardin, M.Bilenky, P.Christova, M.Jack,

L.Kalinovskaya, A.Olchevski, S.Riemann, T.Riemann

`GENTLE/4fan`

D.Bardin, M.Bilenky, J.Biebel, D.Lehner, A.Leike,

A.Olchevski, T.Riemann

`TOPAZ0`

G.Montagna, O.Nicrosini, F.Piccinini, G.Passarino,

`WTO, ZTO`

G.Passarino

$\mathcal{KK}$MC (with `DIZET` EW library, Dubna-Zeuthen)

S.Jadach, B.Ward, Z.Wąs

`KORALW`

S.Jadach, W.Placzek, M.Skrzypek, Z.Wąs, B.F.L.Ward

`ZFITTER` approach:

- one-loop core and general environment based on our own formulae;
- implementation of all world results for higher order QED, QCD and EW RC.

25 free parameters of the SM:

All masses of fundamental particles: $4 \times 3 + 3 = 15$

All mixings, V_{ij} (irrelevant for precision tests at Z-resonance): $= 8$

All coupling constants of fundamental interactions: α and α_S: $= 2$

Number of free parameters in calculations (fits) of LEP observables

Lepton masses and $\alpha(0)$ are known very precise.

Light quark masses and M_W are replaced:

$$\sigma\left(e^+e^- \to \text{hadrons}\right) \quad \to \quad \alpha\left(M_Z^2\right) \to \Delta\alpha_h^{(5)}(M_Z^2)$$

$$\text{then} \quad 1/\tau_\mu \quad \to \quad M_W \quad \to \quad G_F \ \text{ very precise}$$

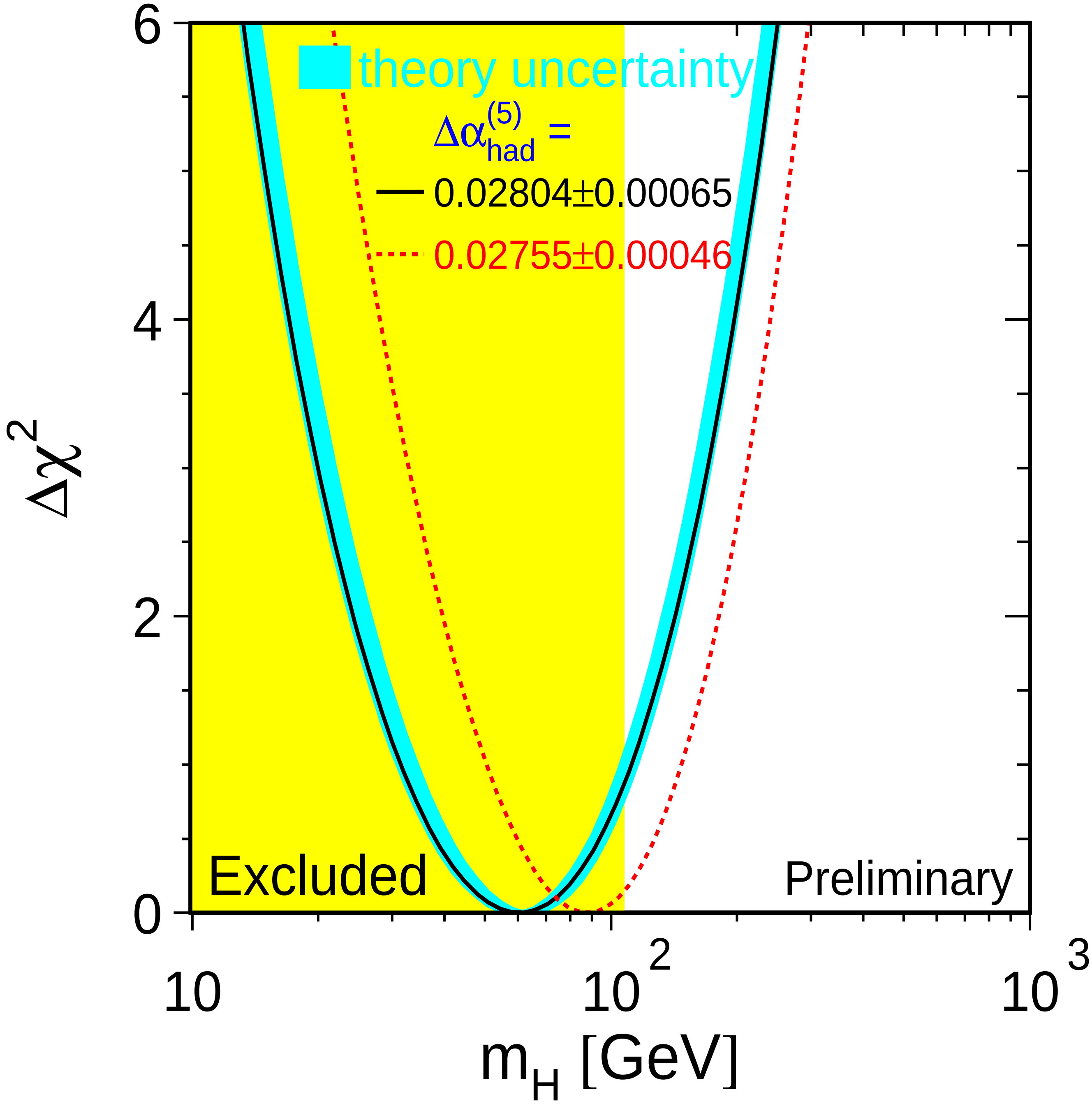
Left with 5 parameters — **LEP1, IPS**:

$$\Delta\alpha_h^{(5)}(M_Z^2) \qquad \alpha_S\left(M_Z^2\right) \qquad m_t \qquad M_Z \qquad \mathbf{M_H}$$

M_Z measured very precise at LEP1 and for the first three parameters an information is available from the other measurements.

Approaching one-parameter fit: $\mathbf{M_H}$

The Blue Band



The running QED coupling $\alpha(s)$

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha_l(s) - \Delta\alpha_h^{(5)}(s) - \Delta\alpha_t(s) - \Delta\alpha^{\alpha\alpha_S}(s)}$$

Hadronic $\Delta\alpha_h^{(5)}(M_Z^2)$ (old and new, ICHEP'2000)

Eidel.&Jegerl.	(exp. only)	0.02804 ± 0.00065	$1/128.8866$
B.Pietrzyk	(exp. only)	0.02755 ± 0.00046	$1/128.9533$
A.D.Martin et al	(exp.+pQCD)	0.02738 ± 0.00020	$1/128.9771$

Typical precision of a **theory driven** analyses:

Davier and Höcker, 1998; *Kühn and Steinhauser, 1998*;

Jegerlehner, 1999.

Leptonic $\Delta\alpha_l(s)$ — from calculations up to three loops.

2-loop: *Källen 1955*; 3-loops: *Steinhauser 1998*.

$$\begin{aligned}\Delta\alpha_l &= 314.97637 \cdot 10^{-4} \\ &= \left[314.18942_{1-\text{loop}} + 0.77616_{2-\text{loop}} + 0.01079_{3-\text{loop}}\right] \cdot 10^{-4}\end{aligned}$$

Example of *saturation* of PT calculations.

Two remaining (for $m_t = 173.8$ GeV and $\alpha_S = 0.119$):

$$\begin{aligned}\Delta\alpha_t(M_Z^2) &= -0.585844 \cdot 10^{-4} \\ \Delta\alpha^{\alpha\alpha_S}(M_Z^2) &= -0.103962 \cdot 10^{-4}\end{aligned}$$

The mixed two-loop $\mathcal{O}\left(\alpha\alpha_S\right)$ correction arising from $t\bar{t}$ loop with gluon exchange is implemented using formulae of *Kniehl 1990*.

Typical scales

LEP1,2 typical scales: $\sqrt{s} = M_Z - 200$ GeV , M_W , M_Z , m_t , M_H ,

all are of the same order and calculations must be **complete**.

One-loop — are!

In real life the notion of m_t^2 *enchanced terms* was introduced:

$$\mathcal{O}\left(G_F m_t^2\right) = \mathcal{O}\left(\frac{\pi}{\sqrt{2}}\frac{\alpha}{s_W^2}\frac{m_t^2}{M_W^2}\right) \quad \text{with} \quad s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$$

$$\frac{m_t^2}{M_W^2} \approx 4 \;\; \text{not so pronounced enchancement!}$$

Since likely, 100 GeV $\leq M_H \leq$ 250 GeV, popular expansions in M_H^2/m_t^2 and m_t^2/M_H^2 have bad convergence.

Complete two-loop EWRC were welcome for LEP1!

Last effort to implement recent results:

Freitas, Hollik, Walter, Weiglein, July 2000.

Coupling constants

QED	$\alpha(0)$	=	$\frac{1}{137}L$	up to $\mathcal{O}\left((\alpha L)^3\right)$
QCD	$\alpha_S\left(M_Z^2\right)$	=	0.119	up to $\mathcal{O}\left(\alpha_S^3\right)$
EW	$\alpha\left(M_Z^2\right)$	=	$\frac{1}{128.9}$	up to $\mathcal{O}\left(\alpha^2\right)$

$$L = \ln\frac{s}{m_e^2} - 1 = 23 \qquad \text{at} \;\; s = M_Z^2, \qquad \alpha L = 0.169$$

Mixed corrections $\mathcal{O}\left(\alpha\alpha_S\right)$, $\mathcal{O}\left(\alpha\alpha_S^2\right)$ needed and implemented.

Veltman's (1975) parameter $\Delta\rho$

$$\delta\rho^{\alpha} = \frac{\alpha}{4\pi s^2_W}\frac{1}{M^2_W}\left[\Sigma_{WW}(M^2_W) - \Sigma_{ZZ}(M^2_Z)\right]$$

$\delta\rho$ terms known in the literature:

$$\delta\rho = \delta\rho^{\alpha} + \delta\rho^{\alpha\alpha_S} + \delta\rho^{\alpha^2} + \delta\rho_L^{\alpha\alpha^2_S}$$

$\mathcal{O}\left(\alpha\alpha_S\right)$ – complete: *Djouadi, 1988; Kniehl, 1990, Djouadi and Gambino, 1993.*

$\mathcal{O}\left(\alpha^2\right)$ – *leading* and *next-to-leading*: *Degrassi et al. 1996–1997*

$$\begin{array}{ccccc} \delta\rho^{\alpha^2} & = & \delta\rho_L^{\alpha^2} & + & \delta\rho_{NL}^{\alpha^2} \\ & & \mathcal{O}\left(\alpha^2 m_t^4/M_W^4\right) & & \mathcal{O}\left(\alpha^2 m_t^2/M_W^2\right) \end{array}$$

The 3-loop correction $\delta\rho^{\alpha\alpha^2_S}$, is known in the leading in m^2_t order: *Avdeev et al, 1994; Chetyrkin, Kühn, Steinhauser, 1995.*

Fanchiotti and Sirlin, 1991 introduced

$$\delta\hat{\rho}^{\alpha} \equiv \frac{\alpha}{4\pi s^2_W}\frac{1}{M^2_W}\left[\Sigma_{WW}(M^2_W) - \Sigma_{ZZ}(M^2_Z)\right]\Bigg|_{\overline{MS},\,\mu=M_Z}$$

For leading in m^2_t terms one performs a *conversion* of couplings $\alpha \to G_F$ and *resummation*: *Consoli, Hollik, Jegerlehner, 1989.*

Resummation of the leading QCD terms, $\delta\rho_L^{G\alpha_S} + \delta\rho_L^{G\alpha^2_S}$ is done: *Halzen and Kniehl, 1991.*

Sirlin's (1980) parameter Δr

$$G_F = \frac{\alpha\pi}{\sqrt{2}M_W^2(1-M_W^2/M_Z^2)}\frac{1}{1-\Delta r}$$

For one-loop QED RC: $G_F = (1.16639 \pm 0.00001)\,\times\,10^{-5}\,\mathrm{GeV}^{-2}$.

With complete two-loop QED RC – *Ritbergen and Stuart, 1998*:

$G_F = (1.16637 \pm 0.00001)\,\times\,10^{-5}\,\mathrm{GeV}^{-2}$.

Δr at one loop
$$\Delta r = \Delta\alpha(M_Z^2) + \frac{c_W^2}{s_W^2}\Delta\hat{\rho}^\alpha + \Delta\hat{r}_{\rm rem}$$

Δr beyond one loop

with the two-loop EW terms $\mathcal{O}\left(G_F^2 m_t^4\right),\ \mathcal{O}\left(G_F^2 m_t^2 M_Z^2\right)$

$$\frac{1}{1-\Delta r} = \frac{1}{\left[1-\Delta\alpha(M_Z^2)-\Delta r^G_{\rm rem}\right]\left[1-\frac{c_W^2}{s_W^2}\delta\hat{\rho}^G\right]}$$

Degrassi, Gambino, Vicini, Sirlin, 1996, 1997.

Irreducible higher order terms are applied by means of

$$\delta\hat{\rho}^G \to \delta\hat{\rho}^{(G)}, \quad \Delta r^G_{\rm rem} \to \Delta r^{(G)}_{\rm rem}$$

where (G) denotes sum of all available corrections and their scale

$$\delta\hat{\rho}^{(G)} = \delta\hat{\rho}^G + \delta\hat{\rho}^{G\alpha_S} + \delta\hat{\rho}^{G\alpha_S^2} + \delta\hat{\rho}^{G^2}$$

$$\Delta r^{(G)}_{\rm rem} = \Delta r^G_{\rm rem} + \Delta r^{G\alpha_S}_{\rm rem} + \Delta r^{G\alpha_S^2}_{\rm rem} + \Delta r^{G^2}_{\rm rem}$$

Leading $\mathcal{O}\left(G\alpha_S^2\right)$: *Chetyrkin, Kühn, Steinhauser, 1995.*

Two-loop Higgs contribution: *van der Bij, Veltman, 1984*

$$\Delta r_L^{H,\alpha^2} = -0.005832\frac{\alpha^2}{\pi^2 s_\theta^4}\frac{M_H^2}{M_W^2}$$

$Z \to f\overline{f}$-decay

The amplitude $\sim \sqrt{\rho_Z^f} I_f^{(3)} \gamma_\mu \left[(1+\gamma_5) - 4|Q_f|\kappa_Z^f s_W^2\right]$

ρ_Z^f and κ_Z^f – *effective couplings*, or *amplitude form factors*.

The form factors comprise one-loop and known two-loop EW (*Degrassi, Gambino, 1999*) and internal QCD corrections $\mathcal{O}\left(\alpha\alpha_S\right)$.

The ratio of effective couplings and *effective weak mixing angles*:

$$
\begin{aligned}
g_Z^f &= \frac{v_f}{a_f} = 1 - 4|Q_f|\left(\kappa_Z^f s_W^2 + I_f^2\right) \\
\sin^2\theta_{\rm eff}^f &= \mathrm{Re}\left(\kappa_Z^f\right) s_W^2 + I_f^2
\end{aligned}
$$

Z decays into a pair of quarks $f = q = u, d, c, s, b$:

$$
\Gamma_f = \Gamma_0\, c_f \left|\rho_Z^f\right| \left[|g_Z^f|^2 R_V^f(M_Z^2) + R_A^f(M_Z^2)\right] + \Delta_{\rm EW/QCD}
$$

Radiator factors, $R_{V,A}^f(M_Z^2)$, describe FSR corrections:

Chetyrkin, Gorishny, Kataev, Kühn, Larin, et al. 1979–1999.

$$
\begin{aligned}
R_B^q(s) &= 1 + \frac{3}{4}Q_q^2\frac{\alpha(s)}{\pi} - \frac{1}{4}Q_q^2\frac{\alpha(s)}{\pi}\frac{\alpha_S(s)}{\pi} + C_0 \\
&\quad + \frac{m_c^2(s)}{s}C_2^{c,B} + \frac{m_b^2(s)}{s}C_2^{b,B} + \frac{m_c^4(s)}{s^2}C_4^{c,B} + \frac{m_b^4(s)}{s^2}C_4^{b,B} \\
&\quad + \text{vector (axial) singlet contribution, B=V,A.}
\end{aligned}
$$

All $C_{0,2,4}^{q,\{V,A\}}$ are presently known up to $\mathcal{O}\left(\alpha_S^3\right)$.

$\Delta_{\rm EW/QCD}$ – *non-factorizable* EW$\otimes$QCD correction: *Czarnecki, Kühn, 1996; Harlander, Seidensticker, Steinhauser, 1998; Fleischer, Jegerlehner, Tentyukov, Veretin, 1999; York-Peng E. Yao, 1999.*

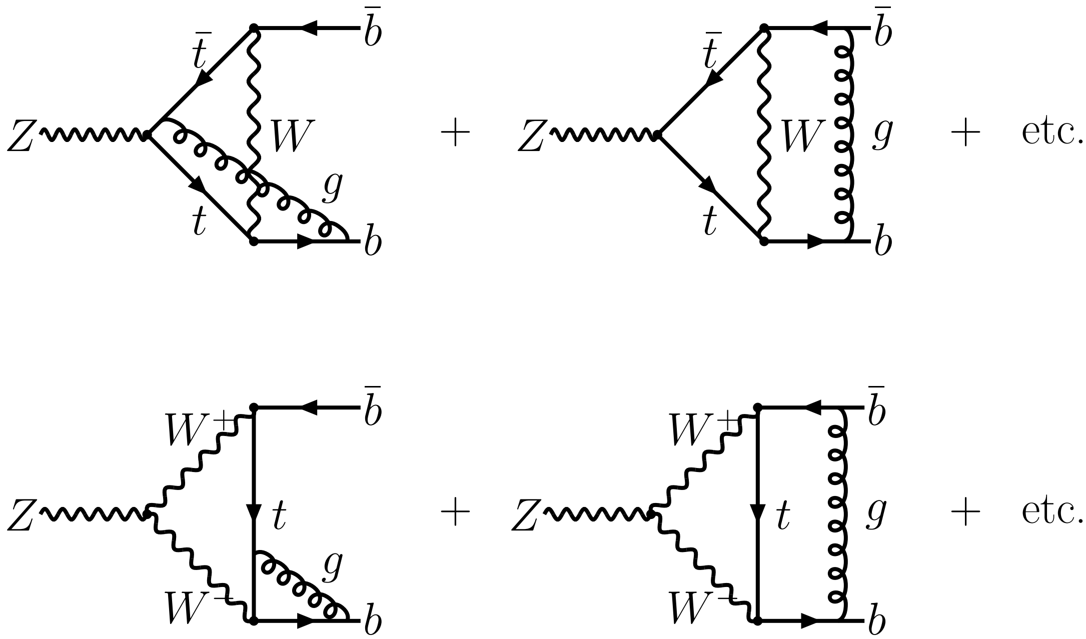
Non-universal corrections for the $Z \to b\bar{b}$

additional m_t-dependent terms absent in the case of light quarks:

Akhundov, DB, Riemann, 1986; Beenakker, Hollik, 1988;

Bernabeu, Pich, Santamaria, 1988, 1991;

and also higher order vertex corrections:



They are into account with the correction τ_b,

$$\tau_b = -2x_t \left[1 - \frac{\pi}{3}\alpha_S(m_t^2) + x_t \tau^{(2)} \left(\frac{m_t^2}{M_H^2} \right) \right]$$

second term: *Fleischer et al. 1992, Buchalla, Buras, 1993;*

Degrassi, 1993; Chetyrkin, Kwiatkowski, Steinhauser, 1993;

last term (only leading in m_t^2): *Barbieri et al., 1992.*

A compact form of $\tau^{(2)}$: *Fleischer, Tarasov, Jegerlehner, 1993.*

NL term $\mathcal{O}\left(G_F^2 m_t^2 M_Z^2\right)$ corrections remained unknown!

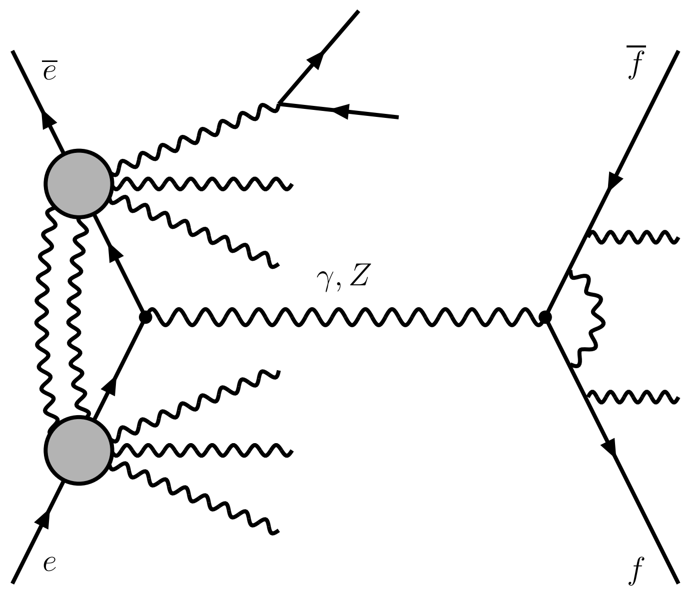
Chain of calculations

Improved Born Approximation amplitudes

→ IBA cross-sections

→ IBA cross-sections ⊕ FSR corrections

$$\rightarrow \sigma^{\mathbf{dec}} = \hat{\sigma}(\mathbf{s}', \mathrm{cuts})$$



$$\rightarrow \sigma^{\mathbf{con}}(\mathbf{s}) = \int \mathbf{dz}\; \mathbf{H_{ISR}}(\mathbf{z}, \mathbf{s})\, \hat{\sigma}(\mathbf{zs})$$

Pure QED ISR corrections

- From complete calculations up to $\mathcal{O}\left(\alpha^2\right)$:

Berends, Burgers, van Neerven, 1988.

- From SF (or FF) methods in the leading log approximation up to $\mathcal{O}\left(\alpha^3 L^3\right)$: *Kuraev and Fadin, 1988; Jadach et al, 1990–1992; Montagna, Nicrosini, Piccinini, 1996; Arbuzov, 1999.*

- QED RC with cuts: *P.Christova, M. Jack, T.Riemann, 1999.*

BOOK HERITAGE

... dozens of book supporting `form` codes written by DB and Giampiero Passarino while working on the book...

The idea was erupted to collect, order, unify and upgrade these codes up to the level of a "computer system":

CalcPHEP

'Calc'ulus of Precision High Energy Physics

being realized at web site: `brg.jinr.ru`

Contributors:

G. Passarino, DB, LK, P.Christova, G.Nanava, A. Andonov

- **Presently available, Phase I:**
  1. **automatic generation and reduction to scalar PV functions of one-loop Feynman diagrams (FD) for all SM $1 \to 2$ decays and $2f \to 2f$ processes (in R_ξ gauge, QCD included);**
  2. **automatic computation of one-loop scalar form factors for the decays $Z(H) \to f\bar{f}$;**
  3. **little help (to be extended soon);**
  4. **already used for $e^+e^- \to t\bar{t}$ (to be presented today).**

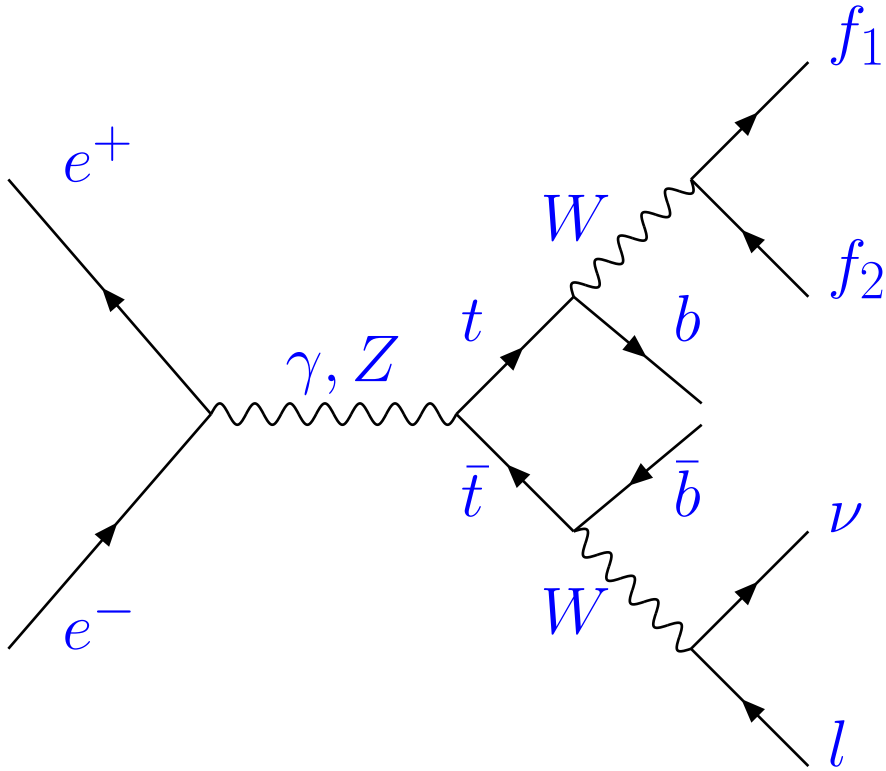
- **Near future, 2001-2002, Phase II:**

1. **extention of computation of one-loop scalar form factors for all SM $1 \to 2$ decays and $2f \to 2f$ processes of experimental interest (REI criterion);**
2. **extention of availability of automatic generation and reduction of FD for all SM $2 \to 2$ processes;**
3. **realisation of the step:**
   from scalar form factors to helicity amplitudes;
4. **realisation of the step:**
   from helicity amplitudes to realistic observables;
5. **solution of the problem of automatic generation of C^{++} codes for numerical calculation of form factors, amplitudes and observables;**
6. **creation of a chain of `form` codes for calculation of one-loop RCs for SM REI $2 \to 3$ processes;**
7. **putting all into the site `brg.jinr.ru`;**
8. **user support of CalcPHEP system; creation of user-friendly environment.**

REI criterion – *Reactions of Experimental Interest:*
initial particles only $e^{\pm}, \gamma$, *muons, neutrinos and partons*.

New calculation for $e^+e^- \to t\bar{t}$; `topfit` project:

DB, LK, G. Nanava; P. Christova, A. Leike, T. Riemann



$\sigma(e^+e^- \to t\bar{t})$ with tops on-mass-shell — ingredient in various approaches: D(Q)PA, MPT

Previous papers:

W. Beenakker, W. Hollik and S.G. van der Marck, *Nucl. Phys.* **B365** (1991) 24; *complete calculation in $\xi = 1$ gauge*

W. Beenakker and W. Hollik, *Phys. Lett.* **B269** (1991) 425;

W. Beenakker, A. Denner and A. Kraft, *Nucl. Phys.* **B410** (1993) 219;

V. Driesen, W. Hollik and A. Kraft, `hep-ph 9603398`.

For MPT see, M. Nekrasow, `hep-ph 0002184`.

The **PURPOSES** are:

- to explicitly control gauge invariance in R_ξ and search for gauge invariant subsets of diagrams

- to compare with the result in the unitary gauge as a cross-check

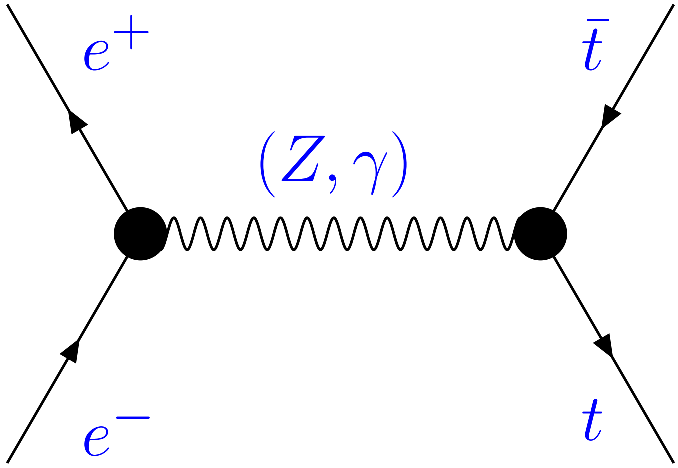
- to present a self-contaned list of results for one-loop amplitude in terms of Passarino-Veltman functions

- to create a FORTRAN code for

IMPROVED **BORN** APPROXIMATION

$$\frac{d\sigma}{dt} \sim |A|^2 = |A_{BORN} + A_{WEAK}|^2$$

AMPLITUDES in L, Q, D - BASIS



$$A \sim$$

$$\left[i\gamma_\mu \left(1+\gamma_5\right) F_L^e\left(s\right) + i\gamma_\mu F_Q^e\left(s\right)\right] \otimes$$

$$\left[i\gamma_\mu \left(1+\gamma_5\right) F_L^t\left(s\right) + i\gamma_\mu F_Q^t\left(s\right) + m_t I D_\mu F_D^t\left(s\right)\right]$$

$$D_\mu = \left(p_{\bar{t}} - p_t\right)_\mu$$

Every form factors in R_ξ gauge could be represented as

$$F^{\xi}_{L,Q,D}(s) = F^{(1)}_{L,Q,D}(s) + F^{add}_{L,Q,D}(s,\xi)$$

First term corresponds to $\xi = 1$ gauge and the second contains all ξ dependence and vanishes for $\xi = 1$.

CHECKED THE CANCELLATION

separately for five subsets of diagrams with

virtual

- $\left\{\gamma \longrightarrow \xi_A\right\}$ – QED
- $\left\{Z, \phi^0 \longrightarrow \xi_Z\right\}$ – VERTICES + WF REN

CLUSTER

- $\left\{H, \phi^0 \longrightarrow \xi_Z\right\}$ **cluster**
- $\left\{W, \phi^\pm \longrightarrow \xi\right\}$ **cluster**

+ self-energies and WW box

- ZZ box

Gauge invariant subsets of **QED** diagrams :

QED cluster

$\gamma\gamma$, $Z\gamma$ boxes

QED bremsstrahlung diagrams

Free of infrared divergences.

The remaining one-loop diagrams form

WEAK corrections.

The total weak amplitude is a sum

of *dressed* γ and Z exchange amplitudes.

BUILDING BLOCKS

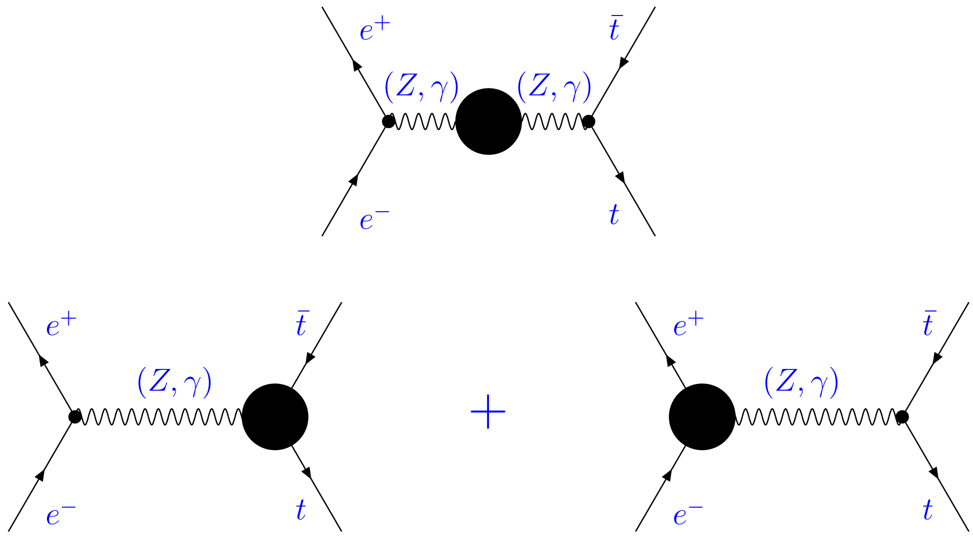
SELF-ENERGIES

VERTICES

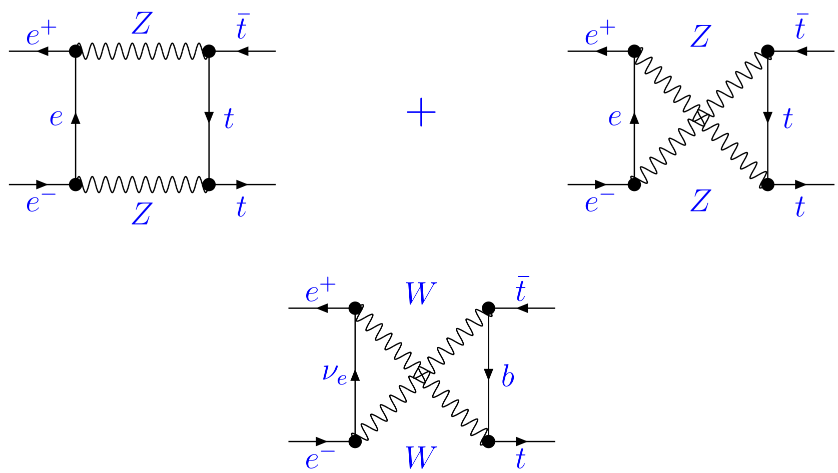
WEAK BOXES (WW and ZZ)

supplied by CalcPHEP

Free of ultraviolet divergences.



EW dressing of propagators and vertices



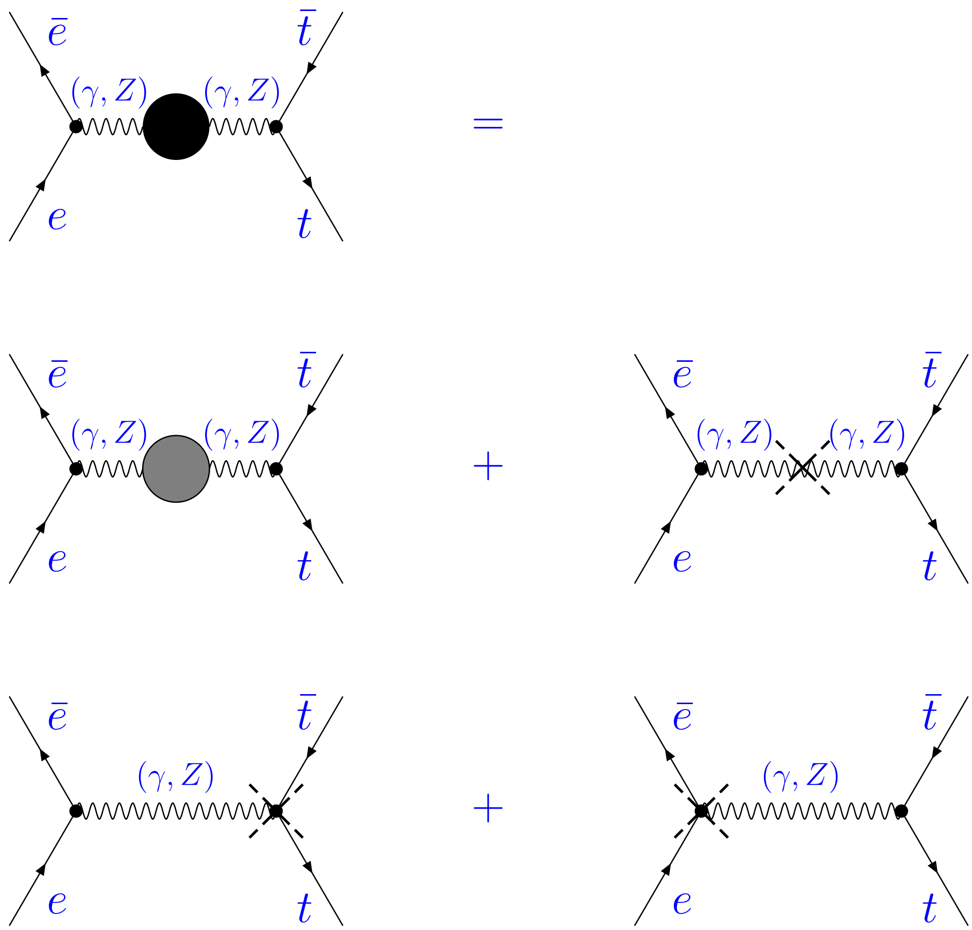
ZZ and WW boxes

Born-like structure of the WEAK AMPLITUDE in terms of LL, QL, LQ, QQ, LD and QD form factors

$$
\begin{aligned}
A_\gamma &= i\frac{4\pi Q_e Q_f}{s}\,\alpha(s)\;\gamma_\mu \otimes \gamma_\mu \\
\mathcal{A}_Z &= i\frac{g^2}{16\pi^2}\,e^2\,4I_e^{(3)}I_t^{(3)}\frac{\chi_Z(s)}{s} \\
&\quad \times\Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5)\;F_{LL}(s,t) \\
&\quad -4|Q_e|s_W^2\gamma_\mu\otimes\gamma_\mu(1+\gamma_5)\;F_{QL}(s,t) \\
&\quad -4|Q_t|s_W^2\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu\;F_{LQ}(s,t) \\
&\quad +16|Q_eQ_t|s_W^4\gamma_\mu\otimes\gamma_\mu\;F_{QQ}(s,t) \\
&\quad +\gamma_\mu(1+\gamma_5)\otimes(-im_tD_\mu)\;F_{LD}(s,t) \\
&\quad -4|Q_e|s_W^2\gamma_\mu\otimes(-im_tD_\mu)\;F_{QD}(s,t)\Big\}
\end{aligned}
$$

$\chi_Z(s)$ – γ/Z **propagator ratio**

Bosonic self-energies and bosonic counter-terms



$$F^{ct}_{LL}(s) = \mathcal{D}_Z(s) - s^2_W \Pi_{\gamma\gamma}(0) + \frac{c^2_W - s^2_W}{s^2_W}\left(\Delta\rho + \Delta\bar{\rho}^{\rm bos}\right)$$

$$F^{ct}_{QL(LQ)}(s) = \mathcal{D}_Z(s) - \left(\Pi_{Z\gamma}(s) + \bar{\Pi}^{\rm bos}_{Z\gamma}(s)\right) - s^2_W \Pi_{\gamma\gamma}(0) - \left(\Delta\rho + \Delta\bar{\rho}^{\rm bos}\right)$$

$$F^{ct,{\rm bos}}_{QQ}(s) = \mathcal{D}^{\rm bos}_Z(s) - 2\left(\Pi^{\rm bos}_{Z\gamma}(s) + \bar{\Pi}^{\rm bos}_{Z\gamma}(s)\right) + c^2_W\left(1 - \frac{M^2_Z}{s}\right)\left[\Pi^{\rm bos}_{\gamma\gamma}(s) - \Pi^{\rm bos}_{\gamma\gamma}(0)\right] - s^2_W \Pi^{\rm bos}_{\gamma\gamma}(0) - \frac{1}{s^2_W}\left(\Delta\rho^{\rm bos} + \Delta\bar{\rho}^{\rm bos}\right)$$

$$F^{ct,{\rm fer}}_{QQ}(s) = \mathcal{D}^{\rm fer}_Z(s) - 2\Pi^{\rm fer}_{Z\gamma}(s) - s^2_W \Pi^{\rm fer}_{\gamma\gamma}(0) - \frac{1}{s^2_W}\Delta\rho^{\rm fer}$$

The term $\sim \left[\Pi^{\rm fer}_{\gamma\gamma}(s) - \Pi^{\rm fer}_{Z\gamma}(s)\right]$ **is extracted from** $F^{ct,{\rm fer}}_{QQ}(s)$ **and shifted to** A^{OLA}_{γ}**.**

$\Delta\bar{\rho}^{\rm bos}$ **and** $\bar{\Pi}^{\rm bos}_{Z\gamma}(s)$ **stand for shifts of bosonic self-energies,** ***G. Passarino*, 1991.**

$$\Delta\rho^{\rm bos} = 4s^2_W\left[\frac{1}{\bar{\epsilon}} - L_\mu(M_W)\right], \quad \bar{\Pi}^{\rm bos}_{Z\gamma}(s) = -2\frac{M^2_W}{s}\left[\frac{1}{\bar{\epsilon}} - L_\mu(M_W)\right]$$

$L_\mu(M^2)$ **denotes the log containing t'Hooft scale** μ**:**

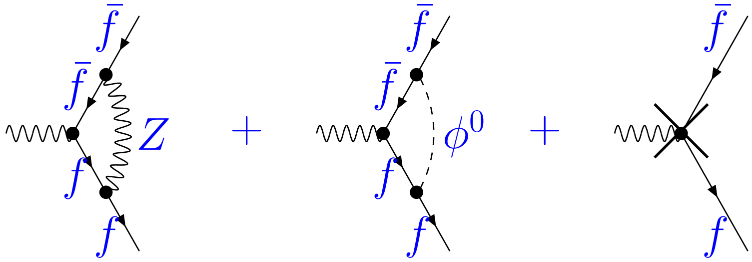
$$L_\mu(M^2) = \ln\frac{M^2}{\mu^2}$$

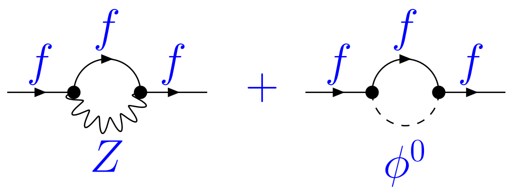
A useful ratio

$$\mathcal{D}_Z(s) = \frac{1}{c^2_W}\frac{\Sigma_{ZZ}(s) - \Sigma_{ZZ}\left(M^2_Z\right)}{M^2_Z - s}$$

SCALAR FORM FACTORS

of Z CLUSTER





Separating out pole contributions $1/\bar{\varepsilon}$ **, we define finite quantities. We note, that if a form factor** $F_A^{ij}(s)$ **has a pole, than the corresponding finite part** $\mathcal{F}_A^{ij}(s)$ **is** μ**-dependent.**

$$F_L^{\gamma Z}(s) = \mathcal{F}_L^{\gamma Z}(s)$$

$$F_Q^{\gamma Z}(s) = \mathcal{F}_Q^{\gamma Z}(s)$$

$$F_D^{\gamma Z}(s) = \mathcal{F}_D^{\gamma Z}(s)$$

$$F_L^{zZ}(s) = -\frac{1}{4} r_{tW} \frac{1}{\bar{\varepsilon}} + \mathcal{F}_L^{zZ}(s)$$

$$F_Q^{zZ}(s) = -\frac{1}{16} \frac{1}{|Q_t|\, s_W^2} r_{tW} \frac{1}{\bar{\varepsilon}} + \mathcal{F}_Q^{zZ}(s)$$

$$F_D^{zZ}(s) = \mathcal{F}_D^{zZ}(s)$$

$$\mathcal{F}_L^{\gamma Z}(s) = \frac{2}{c_W^2} Q_t v_t a_t \Bigg\{ 2\left(2 + \frac{1}{R_Z}\right)$$

$$\times M_Z^2 C_0\big(-m_t^2, -m_t^2, -s; m_t, M_Z, m_t\big)$$

$$-3B_0^F\big(-s; m_t, m_t\big) + 2B_0^F\big(-m_t^2; m_t, M_Z\big) - L_\mu(m_t^2)$$

$$+\frac{1}{r_{tZ}}\Big[B_0^F\big(-m_t^2; m_t, M_Z\big) + L_\mu(M_Z^2) - 1\Big]$$

$$-2\left(1 + 4r_{tZ}\right)\frac{M_Z^2}{4m_t^2 - s} L_{ab}\left(0, m_t, M_Z\right)\Bigg\}$$

$$B_0\big(-s; M_1, M_2\big) = \frac{1}{\bar{\varepsilon}} + B_0^F\big(-s; M_1, M_2\big)$$

Therefore B_0^F also depends of the scale μ

We introduce symbols

$$R_W = \frac{M_W^2}{s} \qquad R_Z = \frac{M_Z^2}{s} \qquad r_{tZ} = \frac{m_t^2}{M_Z^2} \qquad r_{tW} = \frac{m_t^2}{M_W^2}$$

$$r_{tH} = \frac{m_t^2}{M_H^2} \qquad r_{HZ} = \frac{M_H^2}{M_Z^2} \qquad r_{HW} = \frac{M_H^2}{M_W^2}$$

and auxiliary function

$$L_{ab}(M_1, M_2, M_3) = \left(1 + \frac{M_1^2}{M_3^2}\right) M_3^2 C_0\big(0, 0, -s; M_3, M_2, M_3\big)$$

$$-B_0^F\big(-s; M_3, M_3\big) + B_0^F\big(-m_t^2; M_2, M_3\big)$$

$$
\begin{aligned}
\mathcal{F}_Q^{\gamma Z}(s) \;=\; & \frac{1}{4c_W^2}\Bigg\{(v_t-a_t)^2\Bigg[2\Bigg(2\,(1-r_{tZ})+\frac{1}{R_Z}\Bigg)M_Z^2 \\
& \times C_0\big(-m_t^2,-m_t^2,-s;m_t,M_Z,m_t\big) \\
& -3B_0^F\big(-s;m_t,m_t\big)+4B_0^F\big(-m_t^2;m_t,M_Z\big)+L_\mu(M_Z^2) \\
& -r_{tZ}B_{0d}^F\big(-m_t^2;m_t,M_Z\big) \\
& -2\,(1+2r_{tZ})\,M_Z^2B_{0p}\big(-m_t^2;m_t,M_Z\big)-\frac{5}{2}\Bigg] \\
& +2v_ta_tr_{tZ}\Bigg[-4M_Z^2C_0\big(-m_t^2,-m_t^2,-s;m_t,M_Z,m_t\big) \\
& +2\,\big(L_\mu(m_t^2)-L_\mu(M_Z^2)\big) \\
& -2\,(1-r_{tZ})\,B_{0d}^F\big(-m_t^2;m_t,M_Z\big)+1 \\
& -\frac{2}{r_{tZ}}\Bigg((1+2r_{tZ})\,M_Z^2B_{0p}\big(-m_t^2;m_t,M_Z\big)+\frac{1}{2}\Bigg)\Bigg] \\
& +2a_t^2r_{tZ}\Bigg[B_0^F\big(-s;m_t,m_t\big)+L_\mu(M_Z^2) \\
& -r_{tZ}B_{0d}^F\big(-m_t^2;m_t,M_Z\big) \\
& +6M_Z^2B_{0p}\big(-m_t^2;m_t,M_Z\big)-\frac{5}{2}\Bigg] \\
& -4\,\Bigg[\frac{(v_t-a_t)^2}{2}-\big(4v_ta_t-a_t^2\big)\,r_{tZ}\Bigg] \\
& \frac{M_Z^2}{4m_t^2-s}L_{ab}\,(0,m_t,M_Z)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_D^{\gamma Z}(s) \;=\; & \frac{8Q_t}{c_W^2}\frac{1}{4m_t^2-s}\Bigg\{\frac{v_t^2+a_t^2}{2}\Bigg[-4M_Z^2 \\
& \times C_0\big(-m_t^2,-m_t^2,-s;m_t,M_Z,m_t\big) \\
& +\frac{1}{r_{tZ}}\Big[B_0^F\big(-m_t^2;m_t,M_Z\big)+L_\mu(M_Z^2)-1\Big] \\
& +B_0^F\big(-s;m_t,m_t\big) \\
& -2B_0^F\big(-m_t^2;m_t,M_Z\big)-L_\mu(m_t^2)+2 \\
& +6\frac{M_Z^2}{4m_t^2-s}L_{ab}\left(0,m_t,M_Z\right)\Bigg] \\
& +a_t^2\Bigg[2\left(3r_{tZ}-\frac{1}{R_Z}\right)M_Z^2 \\
& \times C_0\big(-m_t^2,-m_t^2,-s;m_t,M_Z,m_t\big) \\
& +B_0^F\big(-m_t^2;m_t,M_Z\big)+L_\mu(M_Z^2)-1 \\
& -r_{tZ}\Big[B_0^F\big(-s;m_t,m_t\big)+L_\mu(m_t^2)-2\Big] \\
& -2\left(2-3\frac{m_t^2}{4m_t^2-s}\right)L_{ab}\left(0,m_t,M_Z\right)\Bigg]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
B_{0d}^F\big(-m_t^2;m_t,M_Z\big) \;=\; & \frac{M_Z^4}{m_t^4}\Bigg[B_0^F\big(-m_t^2;m_t,M_Z\big)+L_\mu(M_Z^2)-1 \\
& +r_{tZ}\left(L_\mu(m_t^2)-L_\mu(M_Z^2)+\frac{1}{2}\right)\Bigg]
\end{aligned}
$$

B_{0p} — **derivative of** B_0 **function.**

EWFF for the process $e^+e^- \to u\bar{u}$

topfit - ZFITTER comparison

Quantity		$E_{\rm cm}$		
		100	**200**	**300**
F_{LL}	$M_W/10$	$12.89587 - i1.84781$	$8.24674 - i10.64677$	$8.98241 - i12.88512$
	M_W	$12.89586 - i1.84781$	$8.24673 - i10.64677$	$8.98241 - i12.88512$
	$10M_W$	$12.89587 - i1.84781$	$8.24673 - i10.64677$	$8.98242 - i12.88512$
ZFITTER		$12.89583 - i1.84786$	$8.24736 - i10.64651$	$8.98370 - i12.88466$
F_{QL}	$M_W/10$	$29.30451 + i3.67334$	$29.38219 + i2.27613$	$31.59712 + i1.59304$
	M_W	$29.30451 + i3.67334$	$29.38218 + i2.27613$	$31.59712 + i1.59304$
	$10M_W$	$29.30451 + i3.67334$	$29.38219 + i2.27613$	$31.59712 + i1.59304$
ZFITTER		$29.30445 + i3.67330$	$29.38216 + i2.27613$	$31.59710 + i1.59304$
F_{LQ}	$M_W/10$	$29.10829 + i3.26972$	$29.48510 + i0.92306$	$31.65836 - i0.89713$
	M_W	$29.10829 + i3.26972$	$29.48509 + i0.92306$	$31.65835 - i0.89713$
	$10M_W$	$29.10829 + i3.26972$	$29.48509 + i0.92306$	$31.65835 - i0.89713$
ZFITTER		$29.10832 + i3.26973$	$29.48512 + i0.92312$	$31.65835 - i0.89711$
F_{QQ}	$M_W/10$	$44.88226 + i8.85688$	$43.31856 + i9.48287$	$44.18773 + i10.25200$
	M_W	$44.88226 + i8.85688$	$43.31854 + i9.48287$	$44.18771 + i10.25200$
	$10M_W$	$44.88226 + i8.85688$	$43.31855 + i9.48287$	$44.18772 + i10.25200$
ZFITTER		$44.88228 + i8.85688$	$43.31854 + i9.48286$	$44.18773 + i10.25196$

SCALAR FORM FACTORS

$$F_{LL}(s,t,u) = \mathcal{F}_L^{zee}(s) + \mathcal{F}_L^{ztt}(s) + \mathcal{F}_{LL}^{ct}(s) + k\ \mathcal{F}_{LL}^{WW}(s,u) + k^{ZZ}\ \mathcal{F}_{LL}^{ZZ}(s,t,u)$$

$$F_{QL}(s,t,u) = \mathcal{F}_Q^{zee}(s) + \mathcal{F}_L^{ztt}(s) + k\ \mathcal{F}_L^{\gamma tt}(s) + \mathcal{F}_{QL}^{ct}(s) + k^{ZZ}\ \mathcal{F}_{QL}^{ZZ}(s,t,u)$$

$$F_{LQ}(s,t,u) = \mathcal{F}_L^{zee}(s) + \mathcal{F}_Q^{ztt}(s) + k\ \mathcal{F}_L^{\gamma ee}(s) + \mathcal{F}_{LQ}^{ct}(s) + k^{ZZ}\ \mathcal{F}_{LQ}^{ZZ}(s,t,u)$$

$$F_{QQ}(s,t,u) = \mathcal{F}_Q^{zee}(s) + \mathcal{F}_Q^{ztt}(s) - \frac{k}{s_W^2}\ \left[\mathcal{F}_Q^{\gamma ee}(s) + \mathcal{F}_Q^{\gamma tt}(s)\right] + \mathcal{F}_{QQ}^{ct}(s) + k^{ZZ}\ \mathcal{F}_{QQ}^{ZZ}(s,t,u)$$

$$F_{LD}(s,t,u) = \mathcal{F}_D^{ztt}(s) + k^{ZZ}\ \mathcal{F}_{LD}^{ZZ}(s,t,u)$$

$$F_{QD}(s,t,u) = \mathcal{F}_D^{ztt}(s) + k\ \mathcal{F}_D^{\gamma tt}(s) + k^{ZZ}\ \mathcal{F}_{QD}^{ZZ}(s,t,u)$$

where

$$k = c_W^2\,(R_Z - 1)$$

$$k^{ZZ} = \frac{(R_Z - 1)}{2c_W^2}$$

IMPROVED BORN APPROXIMATION

$$\sigma_{\gamma\gamma}^{IBA} = Q_t^2 \left(s^2 + 2st + 2t_-^2\right) \left|\alpha(s)\right|^2$$

$$\sigma_{\gamma Z}^{IBA} = 2Q_t Re\Bigg\{\chi\Bigg(2\left[(s+t_-)^2 + sm_t^2\right] F_{LL}(s,t,u)$$

$$+ \left(s^2 + 2st + 2t_-^2\right)$$

$$\times\Big[d_e F_{QL}(s,t,u) + d_t F_{LQ}(s,t,u)$$

$$+ d_e d_t F_{QQ}(s,t,u)\Big]$$

$$-4m_t^2\left(st + t_-^2\right)$$

$$\times\Big[F_{LD}(s,t,u) + d_e F_{QD}(s,t,u)\Big]\Bigg)\alpha^*(s)\Bigg\}$$

$$d_e = -4|Q_e|s_W^2$$

$$d_t = -4|Q_t|s_W^2$$

$$
\begin{aligned}
\sigma_{ZZ}^{IBA} \;=\; & 2\left|\chi\right|^2 \Bigg\{ 8\left(s+t_-\right)^2 \\
& \times\left[\left|F_{LL}\left(s,t,u\right)\right|^2 + d_e F_{LL}\left(s,t,u\right) F_{QL}^*\left(s,t,u\right)\right] \\
& +2\left[\left(s+t_-\right)^2 + t_-^2\right] d_e^2 \left|F_{QL}\left(s,t,u\right)\right|^2 \\
& +4\left[\left(s+t_-\right)^2 + s m_t^2\right] d_t \Big[2F_{LL}\left(s,t,u\right) F_{LQ}^*\left(s,t,u\right) \\
& + d_e F_{LL}\left(s,t,u\right) F_{QQ}^*\left(s,t,u\right) + d_e F_{QL}\left(s,t,u\right) F_{LQ}^*\left(s,t,u\right) \Big] \\
& +\left[s^2 + 2\left(st + t_-^2\right)\right] d_t \Big[2d_t \left|F_{LQ}\left(s,t,u\right)\right|^2 + d_e^2 d_t \left|F_{QQ}\left(s,t,u\right)\right|^2 \\
& +2d_e\left(d_e F_{QL}\left(s,t,u\right) + d_t F_{LQ}\left(s,t,u\right)\right) F_{QQ}^*\left(s,t,u\right)\Big] \\
& -8m_t^2\left(st + t_-^2\right) \Big[\left(2F_{LD}\left(s,t,u\right) + d_e F_{QD}\left(s,t,u\right)\right) F_{LL}^*\left(s,t,u\right) \\
& + d_e \left(F_{LD}\left(s,t,u\right) + d_e F_{QD}\left(s,t,u\right)\right) F_{QL}^*\left(s,t,u\right) \\
& + d_t \left(2F_{LD}\left(s,t,u\right) + d_e F_{QD}\left(s,t,u\right)\right) F_{LQ}^*\left(s,t,u\right) \\
& + d_e d_t \left(F_{LD}\left(s,t,u\right) + d_e F_{QD}\left(s,t,u\right)\right) F_{QQ}^*\left(s,t,u\right) \Big] \\
& -2m_t^2\left(st + t_-^2\right)\left(s - 4m_t^2\right) \Big[2\left|F_{LD}\left(s,t,u\right)\right|^2 \\
& +2d_e F_{LD}\left(s,t,u\right) F_{QD}^*\left(s,t,u\right) + d_e^2 \left|F_{QD}\left(s,t,u\right)\right|^2 \Big] \Bigg\}
\end{aligned}
$$

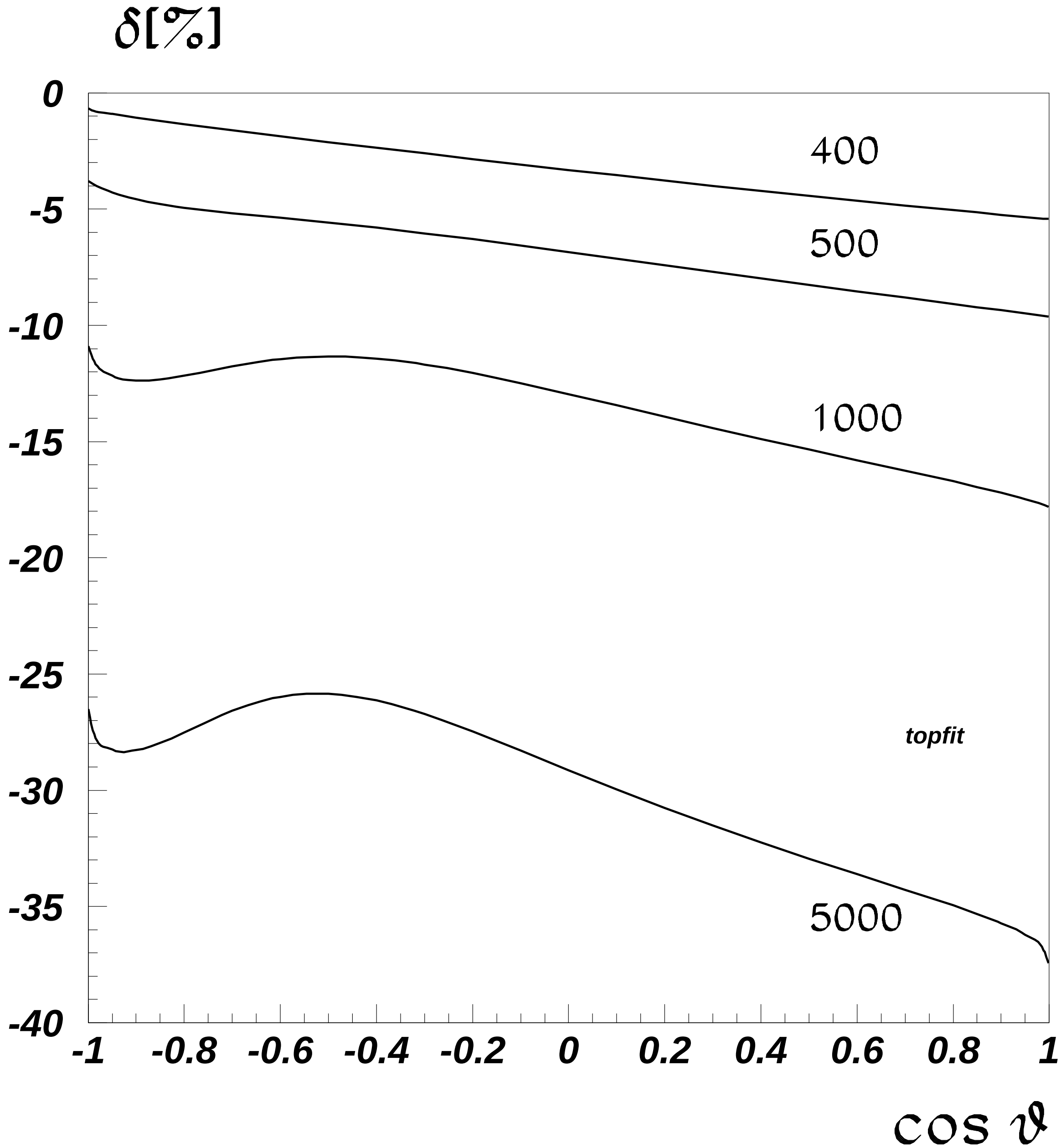
FIRST ROW –ZFITTER (uu channel)

SECONDS ROW – topfit ($m_t = 0.2 GeV$)

THIRD ROW – topfit ($m_t = 173.8 GeV$).

$\sqrt{s}$	100GeV	200GeV	300GeV	400GeV	700GeV	1000GeV
$\cos\theta = -0.9$	47.664652	0.291823	0.169510			
	47.661401	0.291827	0.169515	0.103284	0.035318	0.017203
				0.162579	0.043974	0.018850
$\cos\theta = 0$	59.768387	1.718830	0.695061			
	59.770715	1.718870	0.695072	0.376868	0.117276	0.055870
				0.264874	0.112923	0.054211
$\cos\theta = 0.9$	168.981978	5.954048	2.292260			
	168.991272	5.954166	2.292289	1.222343	0.372903	0.176030
				0.438952	0.293415	0.154784

$$\delta(\sqrt{s}, \cos\vartheta) = \frac{d\sigma^{(1)}(s,t)/dt}{d\sigma^{(0)}(s)/dt} - 1$$

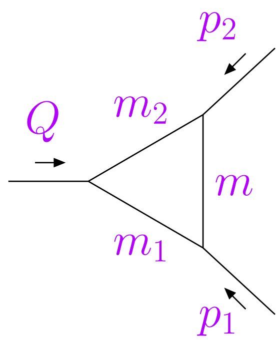


"Algebraic" approach to multiloops (F.Tkachov)

- There is a lot of algebraic structure in Feynman diagrams, and the idea is to exploit it to the maximum.
- That this is indeed possible was discovered in the so-called integration-by-parts, *i-b-p*, algorithm: *F.Tkachov Phys. Lett.* **100B**(1981)65-68.
- It proved to be hugely successful – many well-known NNLO calculations in QCD rely on *i-b-p*.
- In 1996, FT came up with a mathematical result and a scenario to attack arbitrary multiloop diagrams, `hep-ph/9609429`, *NIM* **A389**(1997)309-313.
- It is far from clear whether it is possible to use it even for simplest 2-loop diagrams.
- Worth trying it for a familiar one-loop setting. We did it for scalar form factors $\mathcal{F}^{\gamma Z}_{L,Q,D}(s)$ of Z cluster in $e^+e^- \to t\bar{t}$ (fully massive example).
- Here are some results obtained by *DB, LK and FT*.

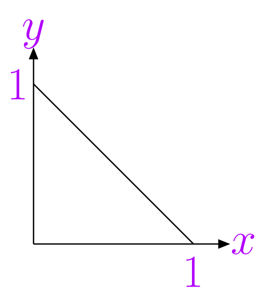
PASSARINO-VELTMAN C_0 FUNCTION

$$i\pi^2\, C_0\left(p_1^2, p_2^2, Q^2; m_1, m, m_2\right) = \mu^{4-n} \int d^n q \frac{1}{d_0 d_1 d_2}$$



$$Q + p_1 + p_2 = 0, \qquad Q^2 = (p_1 + p_2)^2 = -s$$

Simplex in two dimensions



C_0 IN VARIABLES OF SIMPLEX

$$C_0\left(p_1^2, p_2^2, Q^2; m_1, m, m_2\right) = \int_0^1 dx \int_0^{1-x} dy\, \frac{1}{P_{x,y}}$$

$$\begin{aligned} P_{x,y} \; &= \; -x^2\, p_1^2 - y^2\, p_2^2 + x\, y\left(Q^2 - p_1^2 - p_2^2\right) \\ & \quad +x\left(p_1^2 + m_1^2 - m^2\right) + y\left(p_2^2 + m_2^2 - m^2\right) + m^2 \end{aligned}$$

The integration domain (and integrand) are symmetric with respect to interchange of $x \leftrightarrow y$ (and $p_1 \leftrightarrow p_2,\ m_1 \leftrightarrow m_2$).

“LIFTING” OF POLYNOMIAL POWERS

$$P_{x,y}^{k} \;=\; \frac{1}{\boldsymbol{\Delta}}\left[1-\frac{(x+A_x)\,\partial_x+(y+A_y)\,\partial_y}{2(k+1)}\right]P_{x,y}^{k+1}$$

$$P_{x,y}\equiv P_{x,y}\left(m_t,M_Z,m_t\right) \;=\; Q^2xy+m_t^2\left(x+y\right)^2+M_Z^2\left(1-x-y\right)$$

$$P_{x,y}\left(m_t,M_Z,m_t\right)\;\longrightarrow\;\boldsymbol{\Delta}=\frac{\Delta\left(m_t,M_Z,m_t\right)}{\Delta_3},\quad A_{x(y)}=Q^2\frac{M_Z^2}{4\Delta_3}$$

$$P_x\left(m_t,M_Z,0\right) \;=\; M_Z^2x+m_t^2\left(1-x\right)^2$$

$$P_x\left(m_t,M_Z,0\right)\;\longrightarrow\;\boldsymbol{\Delta}=\Delta\left(m_t,M_Z,0\right),\quad A=\frac{M_Z^2-2m_t^2}{2m_t^2}$$

$$P_x\left(m_t,0,m_t\right) \;=\; Q^2x\left(1-x\right)+m_t^2$$

$$P_x\left(m_t,0,m_t\right)\;\longrightarrow\;\boldsymbol{\Delta}=\Delta\left(m_t,0,m_t\right),\quad A=-\frac{1}{2}$$

$$\Delta\left(m_t,M_Z,m_t\right)=-\frac{M_Z^2Q^2(Q^2+4m_t^2-M_Z^2)}{4}$$

$$\Delta\left(m_t,0,m_t\right)=\frac{\left(Q^2+4m_t^2\right)}{4},\qquad \Delta\left(m_t,M_Z,0\right)=-\frac{M_Z^2}{m_t^2}\frac{\left(M_Z^2-4m_t^2\right)}{4}$$

$$\Delta_3=-\frac{1}{4}Q^2(Q^2+4m_t^2),$$ Gram determinant for 3-point function

Vector A and new determinant Δ

$$A^T = \frac{1}{4\Delta_3}\left(\begin{vmatrix} p_1^2+m_1^2-m^2 & Q^2-p_1^2-p_2^2 \\ p_2^2+m_2^2-m^2 & -2p_2^2 \end{vmatrix}, \begin{vmatrix} -2p_1^2 & p_1^2+m_1^2-m^2 \\ Q^2-p_1^2-p_2^2 & p_2^2+m_2^2-m^2 \end{vmatrix}\right)$$

$$\Delta = \frac{1}{8\Delta_3}\begin{vmatrix} -2p_1^2 & p_1^2+m_1^2-m^2 & Q^2-p_1^2-p_2^2 \\ p_1^2+m_1^2-m^2 & 2m^2 & p_2^2+m_2^2-m^2 \\ Q^2-p_1^2-p_2^2 & p_2^2+m_2^2-m^2 & -2p_2^2 \end{vmatrix}$$

Gram determinant

$$\Delta_3 = \begin{vmatrix} -p_1^2 & p_1 \cdot p_2 \\ p_1 \cdot p_2 & -p_2^2 \end{vmatrix}$$

REDUCTON IN n-DIMENSIONS

- Reduction in n-dimensions is an alternative to PV reduction. The scalar form factors are now expressed in terms of untaken integrals over Feynman parameters that appear *inside* and *outside* polynomials (in powers $-k+\varepsilon,\ k=1,0$).

- All x and y, which appear outside, may be eliminated yet in n-dimensions. This is *a reduction* because it reduces expressions to a very limited number of functions which might be termed as *new scalars*.

- Reduction in n-dimensions exploits *i-b-p* and symmetry of *simplex*.

NEW SCALARS

An analog of the usual C_0 in our case is:

$$C_0\left(k,\mu^2;p_1^2,p_2^2,Q^2;m_1,m,m_2\right)=\int_0^1 dx\int_0^{1-x} dy\,P_{x,y}^k\ln\frac{P_{x,y}}{\mu^2}$$

Analogously

$$B_0\left(k,\mu^2;Q^2;M_1,M_2\right)=\int_0^1 dx P_x^k\ln\frac{P_x}{\mu^2}$$

with

$$P_x\ =\ Q^2x\left(1-x\right)+M_1^2x+M_2^2\left(1-x\right)$$

All new scalars, not only B_0, depend on the t'Hooft scale μ.

Short hand notation for new scalars:

$$\hat{L}_0(m_t, M_Z, m_t) \;=\; C_0\left(0, \mu^2; -m_t^2, -m_t^2, -s; m_t, M_Z, m_t\right)$$

$$\hat{L}_0(m_t, 0, m_t) \;=\; B_0\left(0, \mu^2; -s; m_t, m_t\right)$$

$$\hat{L}_0(m_t, M_Z, 0) \;=\; B_0\left(0, \mu^2; -m_t^2; m_t, M_Z\right)$$

$$\hat{L}_1(m_t, M_Z, m_t) \;=\; \frac{1}{M_Z^2} C_0\left(1, \mu^2; -m_t^2, -m_t^2, -s; m_t, M_Z, m_t\right)$$

$$\hat{L}_1(m_t, 0, m_t) \;=\; \frac{1}{M_Z^2} B_0\left(1, \mu^2; -s; m_t, m_t\right)$$

$$\hat{L}_1(m_t, M_Z, 0) \;=\; \frac{1}{M_Z^2} B_0\left(1, \mu^2; -m_t^2; m_t, M_Z\right)$$

for dimensionlessed inverse determinants:

$$k_3\left(m_t, M_Z, m_t\right) \;=\; \frac{4M_Z^2}{s - 4m_t^2 + M_Z^2}$$

$$k_2\left(m_t, 0, m_t\right) \;=\; -\frac{4M_Z^2}{s - 4m_t^2}$$

$$k_2\left(m_t, M_Z, 0\right) \;=\; -\frac{4m_t^2}{M_Z^2 - 4m_t^2}$$

ratios

$$c_W^2 = \frac{M_W^2}{M_Z^2} \qquad r_{tz} = \frac{m_t^2}{M_Z^2} \qquad R_Z = \frac{M_Z^2}{s}$$

and

$$L_\mu(M) = \ln \frac{M^2}{\mu^2}$$

"ONCE LIFTED" EXPRESSIONS

$$F_{L,1}^{\gamma Z} = \frac{Q_t v_t}{c_W^2}\left\{-\frac{1}{r_{tz}}\left[\hat{L}_0(m_t, M_Z, 0) - L_\mu(M_Z) + 1\right]\right.$$

$$-\frac{1}{R_Z}\left(\hat{L}_0(m_t, 0, m_t) - 2\hat{L}_0(m_t, M_Z, m_t) - 1\right)$$

$$\left.+2\hat{L}_0(m_t, M_Z, m_t) - L_\mu(m_t) + 1\right\}$$

"TWICE LIFTED" EXPRESSIONS

$$F_{L,2}^{\gamma Z} = \frac{Q_t v_t}{c_W^2}\left\{-r_{tz}k_3(m_t, M_Z, m_t)\left[4\hat{L}_1(m_t, M_Z, m_t) + \hat{L}_1(m_t, 0, m_t)\right.\right.$$

$$\left.-2\hat{L}_1(m_t, M_Z, 0) + \frac{1}{12}(5 + 2r_{tz})\right]$$

$$-r_{tz}k_2(m_t, 0, m_t)\left(6\hat{L}_1(m_t, 0, m_t) - 2r_{tz}L_\mu(m_t) + \frac{4}{3}r_{tz}\right)$$

$$-k_2(m_t, M_Z, 0)\left(6\hat{L}_1(m_t, M_Z, 0) - 4L_\mu(M_Z) + 2r_{tz}L_\mu(m_t) + \frac{7}{3}\right)$$

$$-\frac{1}{12R_Z^2} + \frac{1}{R_Z}\left(4\hat{L}_1(m_t, M_Z, m_t) - \hat{L}_1(m_t, 0, m_t) + \frac{2}{3} + \frac{1}{2}r_{tz}\right)$$

$$\left.+8\hat{L}_1(m_t, M_Z, 0) + 4\hat{L}_1(m_t, 0, m_t) - 4L_\mu(M_Z) + 2 + \frac{5}{3}r_{tz}\right\}$$

- $F^{\gamma Z}_{L,S}$ – standard approach, analytic expressions in terms of dilogs
- $F^{\gamma Z}_{L,1}$ – "once lifted", numerical computation
- $F^{\gamma Z}_{L,2}$ – "twice lifted", numerical computation

	Re	Im
$F^{\gamma Z}_{L,S}$	1.6298933583	2.32844
$F^{\gamma Z}_{L,1}$	1.6298933589	2.33202
$F^{\gamma Z}_{L,2}$	1.6298933585	2.32840
$F^{\gamma Z}_{Q,S}$	-0.3667251945	0.80847
$F^{\gamma Z}_{Q,1}$	-0.3667251947	0.80831
$F^{\gamma Z}_{Q,2}$	-0.3667251943	0.80848
$F^{\gamma Z}_{D,S}$	$6.689060649 \cdot 10^{-6}$	$1.8099 \cdot 10^{-5}$
$F^{\gamma Z}_{D,1}$	$6.689060655 \cdot 10^{-6}$	$1.8133 \cdot 10^{-5}$
$F^{\gamma Z}_{D,2}$	$6.689060650 \cdot 10^{-6}$	$1.8099 \cdot 10^{-5}$

9-10 digit agreement for Re part

5 digits — for Im part at $k = 1$

OUTLOOK

- **CalcPHEP**:

  1. if one wants to attack a problem of a certain level complexity, not starting just from the previous level, all the intermediate levels have to be unavoidably passed through;
  2. any future code has to be a MC event generator, *collaboration with S.Jadach's group is foreseen;*
  3. complete one-loop corrections for the process $e^+e^- \to f\bar{f}\gamma$ (where f is any fermion, including electron) is a part of the two-loop program for the Z resonance;
  4. one has to accomplish an R&D for Phase III, having in mind an ultimate goal of accessing two-loop precision level control for Z resonance (for GigaZ linac option);
  5. seems within a reach, but needs a lot of time, huge man power and little support...

- `topfit`:

  1. huge competition on the market;
  2. however, *comparison with W. Hollik is underway;*
  3. hope to use results for one-loop RC for 6-fermion process within MPT approach (beyond the scope of this seminar).

- “Algebraic” approach if only for twoloops:
  - undoubtedly works and is useful at one loop;
  - beyond one loop, there are huge algebraic difficulties, explicit solution has never been found;
  - FT currently explores scenarios that look bizarre enough to hold promises for such a complicated problem.
  - are there any chances of success in a foreseeable future? The only way to answer this question is to examine the expertise (FT’s) behind this effort:
    * which is very non-standard for our community;
    * which integrates a non-trivial understanding of the math involved with a considerable skill in algorithm design and software engineering;
    * which has amply proved its worth.
  - we intend to continue digging in this direction, now in the two loop field...

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Transparencies are available from: `/afs/cern.ch/user/b/bardindy/public/DB_LK_CERN_TH_20_10_2000/Part1-5.ps`