One-loop massive QED boxes

(with computer-oriented approach)

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- Linear Collider TESLA - $\sqrt{s} = 500 \div 800 GeV$.
  Top-quark physics - one of topics of TESLA :
  — precise study of m_t and Γ_t at the threshold - $\sqrt{s} \approx 350 GeV$ needed.
  — precise study of the vector and axial vector couplings of t-quark with Z-boson - $\sqrt{s} > 350 GeV$ needed.
  For the process $e^+ + e^- \to t + \bar{t} + \gamma$ we need the observables $\sigma_{TOT}(s)$, $A_{FB}(s)$, $A_{LR}(s)$ et cetera to be calculated precisely, taking into account m_t in radiative corrections.

- Electroweak corrections - see talk of L. Kalinovskaya

- QED corrections: A.Leike, T.Riemann and P.C.

- QED Box-diagram contributions to the amplitude - P.C.

- Form code for a QED-boxes calculation - created in BRG [a] (brg.jinr.ru).

- Results.

[a] BRG – Bardin's Research Group

QED corrections

A gauge invariant subsets of QED diagrams:

- $\gamma\gamma$ and $Z\gamma$ boxes - a bit later

QED bremsstrahlung diagrams

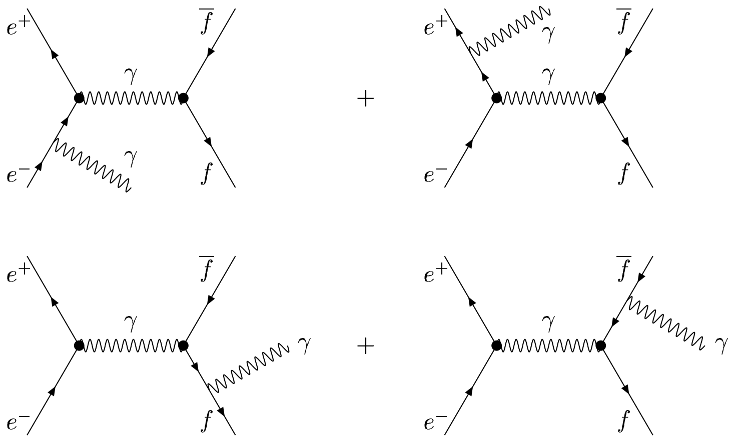

Figure 1: Initial and final state radiation of a real photon

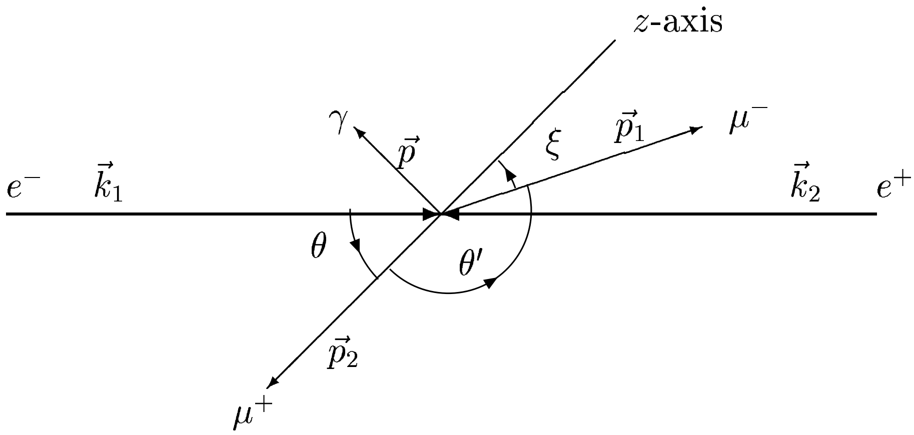

Figure 2: The kinematics of the process with real photon emission.

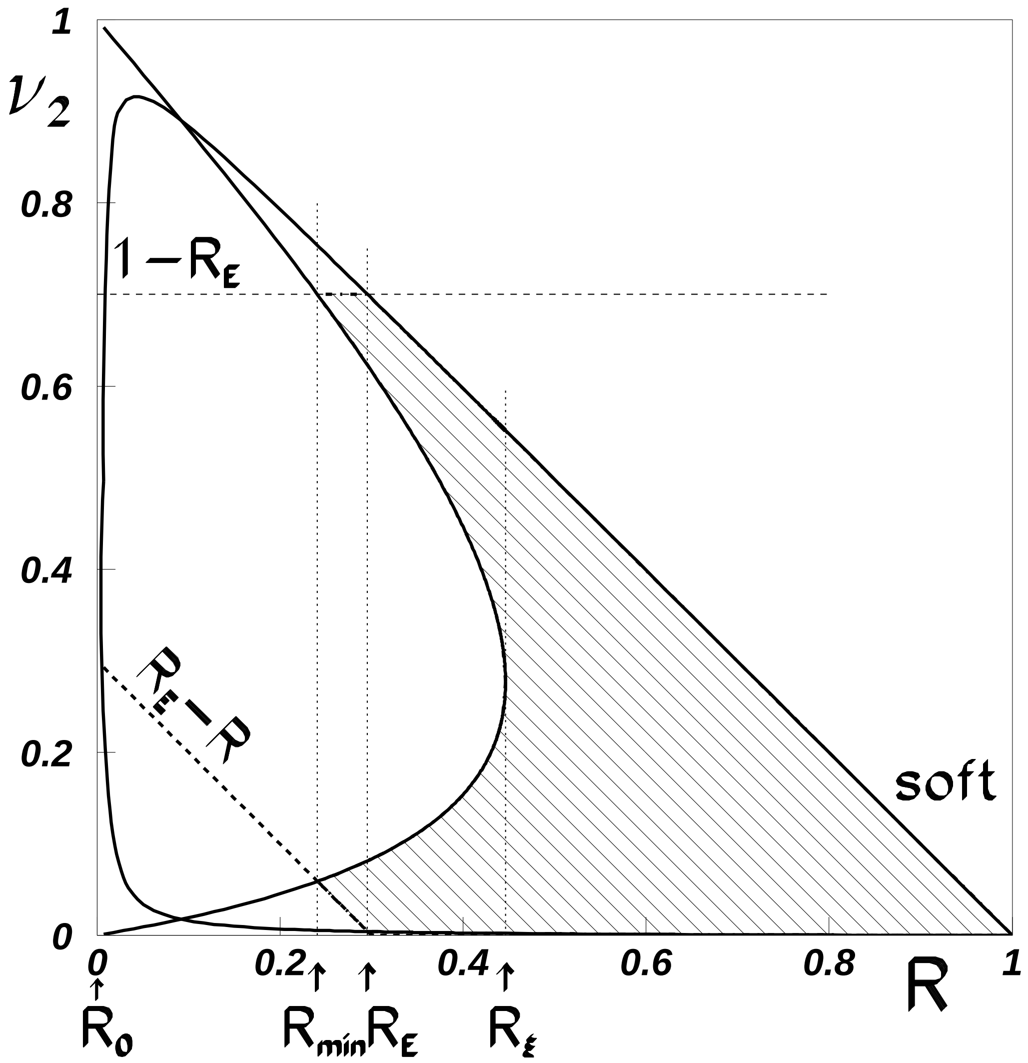

Figure 3: The experimentally available kinematic area of the process is presented using phase-space parametrisation of G.Passarino (Nucl.Phys. **B204**, 1982, p.237).

The variables R and v_2 are

$$R = \frac{s'}{s} \qquad \text{and} \qquad v_2 = \frac{2(p.p_2)}{s},$$

where s' is the invariant mass of the final particles and $p.p_2$ is the scalar product of the momenta of the photon and the final anti-particle.

Soft real photon emission

$$d\sigma^{Soft} = d\sigma^{Born}\,\frac{\alpha}{\pi}\left(\delta^{ISR} + \delta^{FSR} + \delta^{IFI}\right)$$

These three parts are calculated taking into account **the mass of the initial and final particles**.

$$\delta^{ISR}$$

$$\begin{aligned}
= \quad & Q_e^2\Bigg\{\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{(2\omega)^2}{\mu^2}\right)\left[-1 + \left(1 - \frac{2m_e^2}{s}\right)\frac{1}{\beta_e}\ln\frac{1+\beta_e}{1-\beta_e}\right] \\
+ \quad & \frac{1}{\beta_e}\ln\frac{1+\beta_e}{1-\beta_e} + \left(1 - \frac{2m_e^2}{s}\right)\frac{1}{\beta_e}\Bigg[+2Li_2\Big(\frac{1-\beta_e}{1+\beta_e}\Big) \\
- \quad & 2Li_2\Big(1\Big) - \frac{1}{2}\ln^2\frac{1+\beta_e}{1-\beta_e} + 2\ln\frac{1+\beta_e}{1-\beta_e}\ln\frac{1+\beta_e}{2\beta_e}\Bigg]\Bigg\}
\end{aligned}$$

$$\delta^{FSR}$$

$$\begin{aligned}
= \quad & Q_t^2\Bigg\{\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{(2\omega)^2}{\mu^2}\right)\left[-1 + \left(1 - \frac{2m_t^2}{s}\right)\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right] \\
+ \quad & \frac{1}{\beta}\ln\frac{1+\beta}{1-\beta} + \left(1 - \frac{2m_t^2}{s}\right)\frac{1}{\beta}\Bigg[2Li_2\Big(\frac{1-\beta}{1+\beta}\Big) \\
- \quad & 2Li_2\Big(1\Big) - \frac{1}{2}\ln^2\frac{1+\beta}{1-\beta} + 2\ln\frac{1+\beta}{1-\beta}\ln\frac{1+\beta}{2\beta}\Bigg]\Bigg\},
\end{aligned}$$

Here we have

$$\beta_e = \sqrt{1 - \frac{4m_e^2}{s}} \qquad\qquad \beta = \sqrt{1 - \frac{4m_t^2}{s}}.$$

ω is the minimal energy could be measured.

Initial-final interference

Initial-final interference of the soft-photon emission is calculated using the 't Hooft-Veltman trick Nucl.Phys. **B153**, 365, nice presented in the book The Standard Model in the Making of D. Bardin and G. Passarino.

For simplicity here is presented the expression with electron mass neglected where it is possible.

$$\delta^{IFI} = Q_e Q_t \Biggl\{ F(s, c_-) \; - \; F(s, c_+)$$

$$+ \left(\frac{1}{\hat{\varepsilon}} + \ln \frac{(2\omega)^2}{\mu^2} \right) \, s \left[c_- \; J_0(-t, m_e^2, mts) \; - \; c_+ \; J_0(-u, m_e^2, mts) \right] \Biggr\},$$

where

$$c_- = \frac{1}{2}\left(1 - \beta_e \beta \cos\theta\right) \qquad\qquad c_+ = \frac{1}{2}\left(1 + \beta_e \beta \cos\theta\right)$$

and where $\quad F(s, c_-)$

$$= \; -\frac{1}{2}\ln^2\frac{1+\beta}{1-\beta} - \frac{1}{2}\ln^2\frac{m_e^2}{s} - \ln\frac{1+\beta}{1-\beta}\ln\left(1 - \frac{2}{1+\beta}\frac{m_t^2}{s\,c_-}\right)$$

$$- \; \ln^2\left(1 - \frac{2}{1+\beta}\frac{m_t^2}{s\,c_-}\right) - \ln^2\left(-\frac{1}{c_-} + \frac{m_t^2}{s\,c_-^2}\right) - 8Li_2\Bigl(1\Bigr)$$

$$+ \; 2\ln\left(\frac{1}{c_-} - \frac{m_t^2}{s\,c_-^2}\right)\ln\left(1 - \frac{1}{c_-} + \frac{m_t^2}{s\,c_-^2}\right) - 2Li_2\left(\frac{s\,c_-^2}{s\,c_- - m_t^2}\right)$$

$$+ \; 2Li_2\Bigl(1 - \frac{2}{1+\beta}(1 - \frac{m_t^2}{sc_-})\Bigr) - 2Li_2\Bigl(-\frac{(1-\beta)s\,c_-}{(1+\beta)s\,c_- - 2m_t^2}\Bigr).$$

Hard photon initial-final interference effects

Here are presented the numerical effects of m_t.

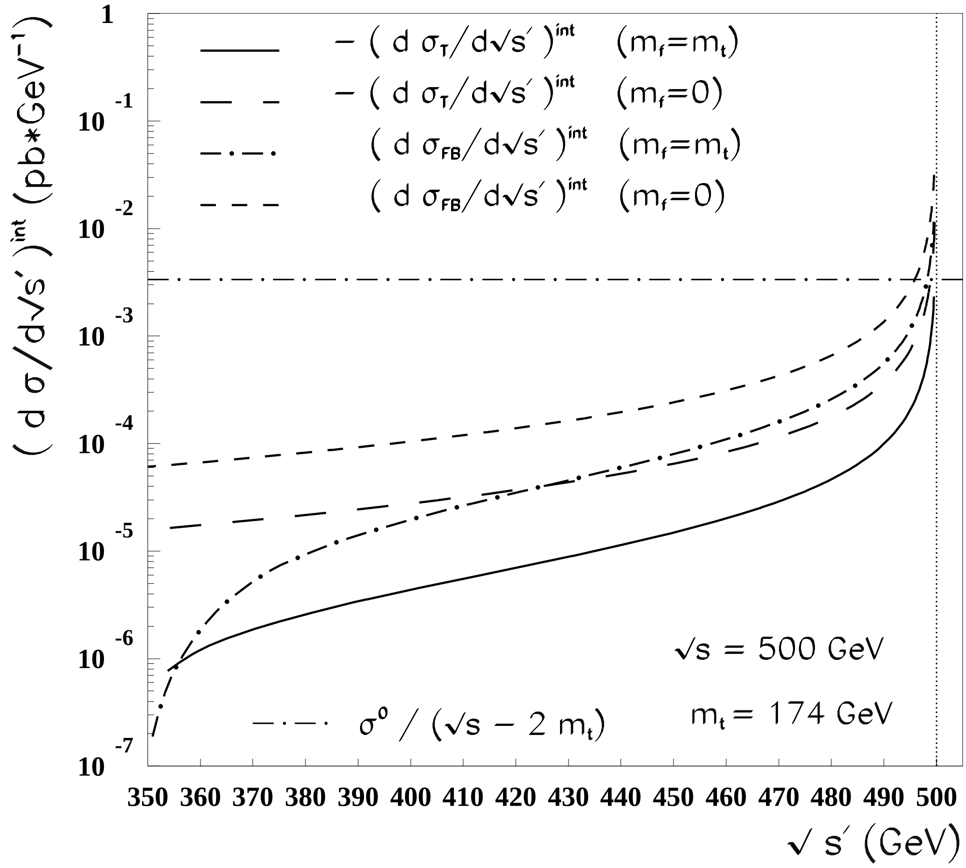

Figure 4: Invariant mass cross section distributions $d\sigma_T^{int}/d\sqrt{s'}$ and $d\sigma_{FB}^{int}/d\sqrt{s'}$, M. Jack et al, hep-ph/0007046.

They are calculated once with mass $m_t = 174 GeV$ of the final particles and, second, neglecting this mass. The main difference between the massive and massless cases is a suppression of cross sections near the threshold $\sqrt{s'} \approx 2m_t$. This suppression is stronger for $d\sigma_{FB}^{IFI}$ than for $d\sigma_{TOT}^{IFI}$. The tree-level cross section, shown for comparison, is $\sigma^0 = 0.51 pb$.

For large invariant masses $\sqrt{s'} \to \sqrt{s}$, the infrared peak clearly dominates. It has to be regularized by soft and virtual (namely, QED box-diagram) corrections.

QED Box-diagram contributions to the amplitude

Gauge invariant photon - Z-boson box-diagrams cluster

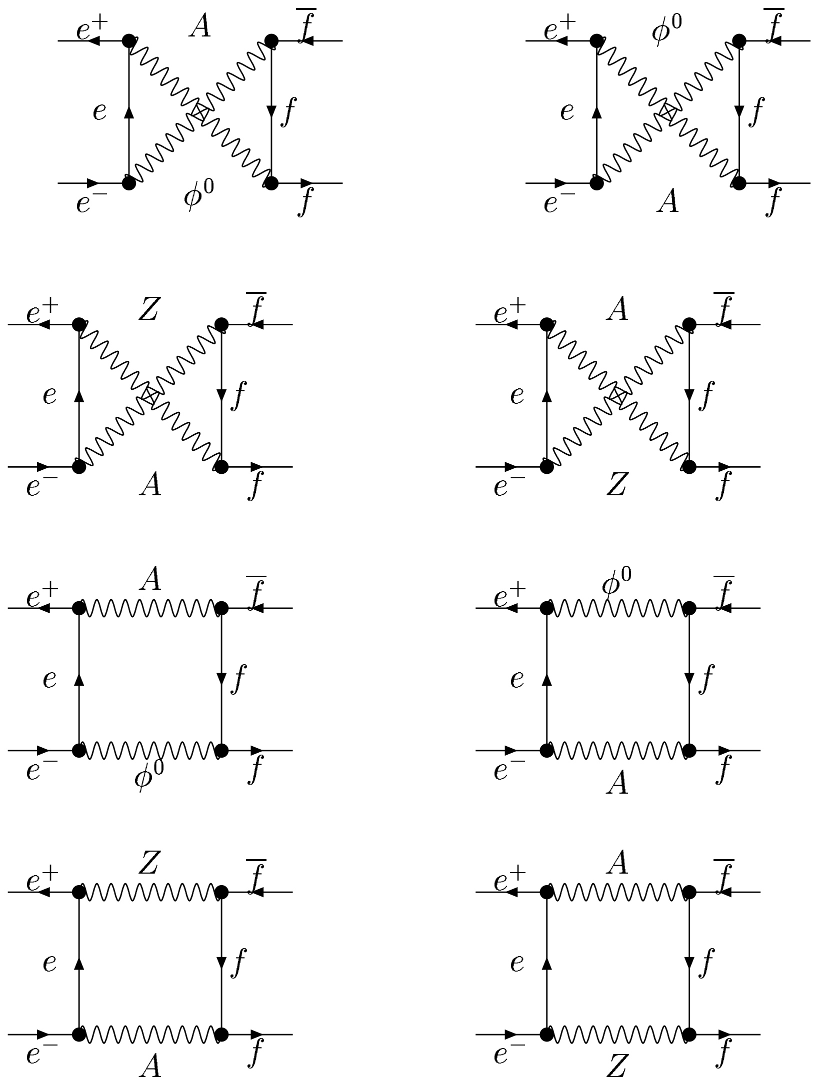

Figure 5: Photon - Z-boson cluster includs diagrams with ϕ^0-boson on the place of Z-boson. The photon - photon cluster looks similar and consists of two diagrams only: one direct and one crossed boxes.

Photon - photon and photon - Z-boson direct box-diagram contributions

$$M^{AA}_{dir} = \frac{1}{(2\pi)^4} \int d^n q$$

$$\bar{v}(p_1) \left[i\, eQ_e\, \gamma^\mu \right] \frac{i(\hat{q}+\hat{p}_1)+m_e}{(q+p_1)^2+m_e^2} \left[i\, eQ_e\, \gamma^\nu \right] u(p_2)$$
$$\frac{1}{q^2} \left[\delta_{\mu\alpha} + (\xi_A^2 - 1) \frac{q_\mu q_\alpha}{q^2} \right]$$
$$\frac{1}{(q+p_1+p_2)^2} \left[\delta_{\nu\beta} + (\xi_A^2 - 1) \frac{(q+p_1+p_2)_\nu (q+p_1+p_2)_\beta}{(q+p_1+p_2)^2} \right]$$
$$\bar{u}(-p_3) \left[i\, eQ_t\, \gamma^\beta \right] \frac{i(\hat{q}-\hat{p}_4)+m_t}{(q-p_4)^2+m_t^2} \left[i\, eQ_t\, \gamma^\alpha \right] v(-p_4),$$

$$M^{AZ}_{dir} = \frac{1}{(2\pi)^4} \int d^n q$$

$$\bar{v}(p_1) \left[i\, eQ_e\, \gamma^\mu \right] \frac{i(\hat{q}+\hat{p}_1)+m_e}{(q+p_1)^2+m_e^2} \left[\frac{i\, g}{2\cos\theta_W} \gamma^\nu \left(v_e + a_e \gamma_5 \right) \right] u(p_2)$$
$$\frac{1}{q^2} \left[\delta_{\mu\alpha} + (\xi_A^2 - 1) \frac{q_\mu q_\alpha}{q^2} \right]$$
$$\left[\frac{1}{(q+p_1+p_2)^2 + M_Z^2} \left(\delta_{\nu\beta} + \frac{(q+p_1+p_2)_\nu (q+p_1+p_2)_\beta}{M_Z^2} \right) \right.$$
$$\left. - \frac{(q+p_1+p_2)_\nu (q+p_1+p_2)_\beta}{M_Z^2 \left((q+p_1+p_2)^2 + \xi_Z^2 M_Z^2 \right)} \right]$$
$$\bar{u}(-p_3) \left[\frac{i\, g}{2\cos\theta_W} \gamma^\beta \left(v_t + a_t \gamma_5 \right) \right] \frac{i(\hat{q}-\hat{p}_4)+m_t}{(q-p_4)^2+m_t^2} \left[i\, eQ_t \gamma^\alpha \right] v(-p_4).$$

Form code for a QED-boxes calculation

```
#include Declar.h
#call Globals{dummy}
.global
#procedure direct(iu,id,fu,fd,vs,vn)
g bd0'iu''id''fu''fd''vs''vn' =
     tlo*vert('vn','iu','id',mu,ii)
        *(i_*gd(ii,qfo)  +pm('id'))*pr('id',qfo)
        *vert('vs','iu','id',nu,ii)*tro
    *tle*vert('vs','fd','fu',be,jj)
        *(i_*gd(jj,qfed) +pm('fd'))*pr('fd',qfed)
        *vert('vn','fd','fu',al,jj)*tre
    *pr('vn',al,mu,q)*pr('vs',be,nu,qvs)*int;
#endprocedure
*
#procedure crossed(iu,id,fu,fd,vs,vn)
g bc0'iu''id''fu''fd''vs''vn' =
     tlo*vert('vn','iu','id',mu,ii)
        *(i_*gd(ii,qfo)   +pm('id'))*pr('id',qfo)
        *vert('vs','iu','id',nu,ii)*tro
    *tle*vert('vn','fd','fu',be,jj)
        *(-i_*gd(jj,qfec) +pm('fd'))*pr('fd',qfec)
        *vert('vs','fd','fu',al,jj)*tre
    *pr('vn',be,mu,q)*pr('vs',al,nu,qvs)*int;
#endprocedure
#call CalcBoxnc{12|12|13|13}
.end
```

Procedure of QED-boxes calculation

```
#procedure CalcBoxnc(iu,id,fu,fd)
*---------------------------------
#call ClasterBOXnc{'iu'|'id'|'fu'|'fd'}
#call FeynmanRules{dummy}
#call Gammaright{dummy}
.sort
#call Boxprereduction{'iu'|'id'|'fu'|'fd'}
#call Reduction{dummy}
#call Diracizing{dummy}
#call Diraceq{'iu'|'id'|'fu'|'fd'}
.sort
#call Diracizing{dummy}
#call Diraceq{'iu'|'id'|'fu'|'fd'}
.sort
#call Diracizing{dummy}
#call Scalprod{dummy}
#call Diraceq{'iu'|'id'|'fu'|'fd'}
#call Epsilon{dummy}
#call Sing{dummy}
#call Scalarizing{dummy}
#call Masses{'iu'|'id'|'fu'|'fd'}
id em=0;
print;
.store
save box_clNC'fu''fd'.sav;
.end
#endprocedure
```

Results: contributions of the QED box-diagrams to the amplitude

Born amplitude for comparison $M^{Born} =$

$$+ie^2 Q_e Q_t \frac{1}{s} \; \bar{v}(p_1)\gamma_\mu u(p_2) \otimes \bar{u}(-p_3)\gamma_\mu v(-p_4)$$

$$+i\frac{g^2}{4\cos^2\theta_W}\frac{1}{s-M_z^2} \; \bar{v}(p_1)\gamma_\mu(v_e+a_e\gamma_5)u(p_2) \otimes$$
$$\bar{u}(-p_3)\gamma_\mu(v_t+a_t\gamma_5)v(-p_4).$$

We will use the notation ... $\otimes$ ... instead of

$$\bar{v}(p_1)...u(p_2) \otimes \bar{u}(-p_3)...v(-p_4).$$

Photon-photon boxes $M^{Box}_{AA} = i\frac{e^4}{64\pi^2}Q_e^2 Q_t^2 \Big\{$

$$\left[\hat{D}\otimes I\right] F^{AA}_{DI} \; + \; \left[\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_+ + \gamma_\mu\gamma_- \otimes \gamma_\mu\gamma_-\right] F^{AA}_{eq}$$
$$+ \; \left[\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_- + \gamma_\mu\gamma_- \otimes \gamma_\mu\gamma_+\right] F^{AA}_{op} \Big\},$$

Photon - Z-boson boxes $M^{Box}_{AZ} = i\frac{e^2 g^2}{128\pi^2 c_\theta^2} Q_e Q_t \Big\{$

$$\left[\hat{D}\gamma_+ \otimes I\right](v_e+a_e) F^{AZ}_{DI+} \; + \; \left[\hat{D}\gamma_- \otimes I\right](v_e-a_e) F^{AZ}_{DI-}$$
$$+ \; \left[\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_+\right](v_e+a_e) F^{AZ}_{LL} \; + \; \left[\gamma_\mu\gamma_- \otimes \gamma_\mu\gamma_-\right](v_e-a_e) F^{AZ}_{RR}$$
$$+ \; \left[\gamma_\mu\gamma_+ \otimes \gamma_\mu\gamma_-\right](v_e+a_e) F^{AZ}_{LR} \; + \; \left[\gamma_\mu\gamma_- \otimes \gamma_\mu\gamma_+\right](v_e-a_e) F^{AZ}_{RL} \Big\},$$

where $\gamma_+ = 1+\gamma_5$ and $\gamma_- = 1-\gamma_5$; $\quad D = p_3 - p_4$.

Photon-photon boxes contribution has only three different structures but the independant coefficient functions are two.

$$\begin{aligned} F_{DI}^{AA} &= F^{AA}(s,t) \quad - \quad F^{AA}(s,u), \\ F_{eq}^{AA} &= G^{AA}(s,t,u) \quad \text{and} \quad F_{op}^{AA} = -G^{AA}(s,u,t). \end{aligned}$$

$$\mathbf{F^{AA}(s,t)} = \frac{i\, m_t}{st+(t-m_t^2)^2} \left\{ \frac{1}{st+(t-m_t^2)^2} \Big[-(t-m_t^2)^3 J_{dir}^{AA}(s,t) \right.$$

$$+ ts^2 C_0(0,0,-s;0,m_e,0) + ts(s-2m_t^2) C_0(-m_t^2,-m_t^2,-s;0,m_t,0) \Big]$$

$$- \quad \frac{2\,t}{s-4m_t^2} \Big[2m_t^2 C_0(-m_t^2,-m_t^2,-s;0,m_t,0) + B_0^F(-m_t^2;m_t,0)$$

$$\left. - \quad B_0^F(-s;0,0) \Big] + \frac{2\,t}{t-m_t^2} \Big[B_0^F(-t;0,m_t) - B_0^F(-m_t^2,m_t,0) \Big] \right\},$$

$$\mathbf{G^{AA}(s,t,u)} =$$

$$- \quad 2 \ \frac{t-m_t^2}{s} \Big[J_{dir}^{AA}(s,t) - 2C_0(-m_t^2,0,-t;m_t,0,0) \Big]$$

$$+ \quad 2 \ \frac{u-m_t^2}{s} \Big[J_{cro}^{AA}(s.u) - 2C_0(-m_t^2,0,-u;m_t,0,0) \Big]$$

$$+ \frac{1}{tu-m_t^4} \left\{ \quad (t-m_t^2) \Big[-t + (t+m_t^2) \frac{(t-m_t^2)^2}{tu-m_t^4} \Big] \; J_{dir}^{AA}(s,t) \right.$$

$$+ \ (u-m_t^2) \Big[u - (t+m_t^2) \frac{(u-m_t^2)^2}{tu-m_t^4} \Big] \; J_{cro}^{AA}(s,u)$$

$$
\begin{aligned}
- & \quad (t-u)\left(1+\frac{s(t+m_t^2)}{tu-m_t^4}\right)\Big[sC_0(0,0,-s;0,m_e,0) \\
& \qquad + (s-4m_t^2)C_0(-m_t^2,-m_t^2,-s;0,m_t,0)\Big] \\
+ & \quad 2\,m_t^2\,s\left(1+\frac{(s-4m_t^2)(t-m_t^2)}{tu-m_t^4}\right)C_0(-m_t^2,-m_t^2,-s;0,m_t,0) \\
+ & \quad 2\,(t-m_t^2)\Big[B_0^F(-t;0,m_t)-B_0^F(-s,0,0)\Big] \\
+ & \quad 2\,m_t^2\left(1+\frac{2t}{t-m_t^2}\right)\Big[B_0^F(-t;0,m_t)-B_0^F(-m_t^2,m_t,0)\Big] \\
+ & \quad 2\,m_t^2\left(1-\frac{2u}{u-m_t^2}\right)\Big[B_0^F(-u;0,m_t)-B_0^F(-m_t^2,m_t,0)\Big]\Bigg\}.
\end{aligned}
$$

where $st+(t-m_t^2)^2 = su+(u-m_t^2)^2 = -(tu-m_t^4)$

Here we introduse the following auxiliary functions instead of $D_0(0,0,-m_t^2,-m_t^2,-s,-t;0,0,0,m_t)$:

$$
\begin{aligned}
J_{dir}^{AA}(s,t) &= 2C_0(-m_t^2,0,-t;m_t,0,0) \\
&+ s\,D_0(0,0,-m_t^2,-m_t^2,-s,-t;0,0,0,m_t) \\
&= \int_0^1 dx\,\frac{\ln\frac{x(1-x)(-t)+xm_e^2+(1-x)m_t^2}{-s}}{x(1-x)(-t)+xm_e^2+(1-x)m_t^2} \\
&+ \frac{1}{t-m_t^2}\Big[\frac{1}{2}\ln^2\frac{1+\beta}{2}+\ln\frac{1+\beta}{2}\ln\frac{-s}{m_t^2}+3Li_2(1) \\
&- 2Li_2\Big(\frac{1+\beta}{2}\Big)-Li_2\Big(\frac{2}{1+\beta}\Big)-Li_2\Big(\frac{1-\beta}{2}\Big)\Big],
\end{aligned}
$$

$$
J_{cro}^{AA}(s,u) = J_{dir}^{AA}\,(t\Rightarrow u).
$$

Photon- Z-boson boxes contribution has only six different structures and the same number independant coefficient functions. They are linear combinations of of the coupling constants v_t and a_t.

$$
\begin{aligned}
F^{AZ}_{DI+} &= (v_t + a_t)\, F^{AZ}(s,t,u) - (v_t - a_t)\, F^{AZ}(s,u,t) \\
F^{AZ}_{DI-} &= (v_t - a_t)\, F^{AZ}(s,t,u) - (v_t + a_t)\, F^{AZ}(s,u,t) \\
F^{AZ}_{LL} &= (v_t + a_t)\, G^{AZ}(s,t,u) - (v_t - a_t)\, H^{AZ}(s,u,t) \\
F^{AZ}_{RR} &= (v_t - a_t)\, G^{AZ}(s,t,u) - (v_t + a_t)\, H^{AZ}(s,u,t) \\
F^{AZ}_{LR} &= (v_t + a_t)\, H^{AZ}(s,t,u) - (v_t - a_t)\, G^{AZ}(s,u,t) \\
F^{AZ}_{RL} &= (v_t - a_t)\, H^{AZ}(s,t,u) - (v_t + a_t)\, G^{AZ}(s,u,t)
\end{aligned}
$$

$$
\begin{aligned}
F^{AZ}(s,t,u) &= F^{AZ}_{main}(s,t) + F^{AZ}_{auxi}(s,u), \\
G^{AZ}(s,t,u) &= G^{AZ}_{main}(s,t) + G^{AZ}_{auxi}(s,u), \\
H^{AZ}(s,t,u) &= H^{AZ}_{main}(s,t) + H^{AZ}_{auxi}(s,u)
\end{aligned}
$$

$$
\mathbf{F^{AZ}_{main}(s,t)} = \frac{i\, m_t}{st + (t - m_t^2)^2} \left\{ + \frac{(t - m_t^2)^2}{s} J^{AA}_{dir}(s,t) \right.
$$

$$
\begin{aligned}
- &\quad 2tC_0(0,-m_t^2,-t;0,M_z,m_t) + (t - m_t^2)\left[\left(-2 + \frac{M_z^2}{s}\right) J^{AZ}_{dir}(s,t)\right. \\
- &\quad \left. 4C_0(0,-m_t^2,-t;0,M_z,m_t)\right] + \frac{2(s - M_z^2)(t - m_t^2)}{st + (t - m_t^2)^2}\left[t\, J^{AZ}_{dir}(s,t)\right. \\
+ &\quad 2tC_0(0,-m_t^2,-t;0,M_z,m_t) - (t - m_t^2)C_0(0,0,-s;M_z,0,0) \\
&\qquad \left. - (t + m_t^2)C_0(-m_t^2,-m_t^2,-s;M_z,m_t,0)\right] \ldots
\end{aligned}
$$