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QED Radiative Corrections within the CalcPHEP Project

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OUTLINE

1. Introduction

CalcPHEP system: 1. Analytic calculations by **form3**, 2. Numeric calculations by **fortran**.
One-loop analytic calculations give the amplitudes of the processes in terms of **Form Factors**.
The QED parts of these **Form Factors** are the **Virtual** contribution in the QED corrections.

Level-1, **form3**

BffRen eeffRen

$\Downarrow$ $\Downarrow$

Virtual

$\Downarrow$

BffBrem $\Longrightarrow$ **Soft** $\Longrightarrow$ / **Hard** $\Longrightarrow$ **QED Amplitudes** M_{ij} $\Longrightarrow$ **1.** $\sum | M_{ij} |^2$ **2. Integration over phase space parameters** $\Longrightarrow$ $s2n.f$

Level-2, **fortran**

2. The basic procedures needed

FeynmanRules.prc - to create the Amplitude to be treated.
GammaRight.prc - to move the γ_5 to right.
Pulling.prc and Pullingc.prc - to move momenta $\hat{p}_i$ of the external fermions close to corresponding bispinors.
Diraceq.prc and Diraceqc.prc - to realize corresponding Dirac equations.

3. One-loop form-factors of the processes

Here we consider only decays of Z-, W-, and H- bosons to fermions.

The **Virtual** contribution in the QED amplitude of Z decay in terms of **Form Factors** is:

$$M^{1-loop}_{Zff} = \frac{ig}{2\cos\theta_W}\,\epsilon^Z_\mu\,\frac{g^2}{16\pi^2}\,\bar{U}(k_1)\Big(\quad i\,I^{(3)}_f\,\gamma^\mu\,(1+\gamma_5)\;\; \mathbf{F^Z_L} - i\,2\,Q_f\,\sin^2\theta_W\,\gamma^\mu\;\;\mathbf{F^Z_Q} + I^{(3)}_f m_f\,({k_2}^\mu - {k_1}^\mu)\;\mathbf{F^Z_D}\;\Big)\;V(k_2).$$

The **Virtual** contribution in the QED amplitude of W decay is:

$$M^{1-loop}_{Wff} = \frac{ig}{2\sqrt{2}}\,\epsilon^W_\mu\,\frac{g^2}{16\pi^2}\,\bar{U}(k_1)\Big(\,i\,\gamma^\mu\,(1+\gamma_5)\,\mathbf{F^W_L}\,\Big)\;V(k_2).$$

The **Virtual** contribution in the QED amplitude of H decay is:

$$M^{1-loop}_{Hff} = -\,\frac{igm_f}{2M_W}\,\frac{g^2}{16\pi^2}\,\bar{U}(k_1)\Big(\,\mathbf{F^H_S}\,\Big)\;V(k_2).$$

How CalcPHEP system calculates these **Form Factors** was shown **in the previous talk**.
The procedure **BffdecayQED.prc** loads the QED parts of fermionic boson decay amplitudes.

4. Soft Bremsstrahlung

The **Soft** bremsstrahlung amplitude is proportional to **Born** amplitude. Here we present the corresponding expressions for Z-, W-, and H- decays.

$$M_{Zff}^{Soft} = e\,Q_f \left(\frac{k_1 \cdot \epsilon^\gamma}{k_1 \cdot p} - \frac{k_2 \cdot \epsilon^\gamma}{k_2 \cdot p} \right) M_{Zff}^{Born}, \quad M_{Hff}^{Soft} = e\,Q_f \left(\frac{k_1 \cdot \epsilon^\gamma}{k_1 \cdot p} - \frac{k_2 \cdot \epsilon^\gamma}{k_2 \cdot p} \right) M_{Hff}^{Born}$$

$$M_{Wff}^{Soft} = e \left(Q_u \frac{k_1 \cdot \epsilon^\gamma}{k_1 \cdot p} - Q_d \frac{k_2 \cdot \epsilon^\gamma}{k_2 \cdot p} - Q_W \frac{(k_1 + k_2) \cdot \epsilon^\gamma}{(k_1 + k_2) \cdot p} \right) M_{Wff}^{Born}$$

Let us write the real photon momentum in the form $p = p^0 n$. Then $\vec{n} \cdot \vec{k}_2 = \mid \vec{k}_2 \mid \cos\theta$. The **Soft** photon has very small energy $0 \leq p^0 \leq \omega$. Therefore we can write $M^{Soft} = \frac{1}{p^0} F(k_1, k_2, n)\, M^{Born}$

$$\Longrightarrow \Gamma^{Soft} = \Gamma^{Born} \frac{1}{(2\pi)^2} \int\limits_{m_\gamma}^{\omega} \frac{(p^0)^2 dp^0}{2p^0} \int\limits_{-1}^{1} d\cos\theta\, \frac{1}{(p^0)^2} F^2(k_1, k_2, n)$$

The integration over p^0 gives the well known **infrared divergence** in terms of $ln\frac{2\omega}{m_\gamma}$. This treatment and integration over $\cos\theta$ using some table of integrals is realized in our procedures **NCdecaySoft.prc** and **WdecaySoft.prc**. So we obtain Γ_{Zff}^{Soft}, Γ_{Wff}^{Soft} and Γ_{Hff}^{Soft}.

Scheme of calculation of **Soft Bremsstrahlung**

BornBff.prc

Born amplitude M^{Born}

BremBff.prc

Bremsstrahlung amplitude M^{Brem}

↘ ↗

$$\mathbf{M}^{\mathbf{Brem}} \stackrel{p=0}{\Longrightarrow} \mathbf{M}^{\mathbf{Soft}}$$

zeroSoftBff.prc

proportionality of **Soft** to **Born** amplitudes

↗ ↘

NCdecaySoft.prc

Table of integrals over $\cos\theta$

$$\sum \mid \mathbf{M}^{\mathbf{Soft}} \mid^{\mathbf{2}}$$

WdecaySoft.prc

Difference between **NCdecaySoft.prc** and **WdecaySoft.prc**

$$\int_{-1}^{1} d\cos\theta \, [\,]$$

NCdecaySoft.prc

$$[\,] = \frac{k_1^2}{(k_1\cdot n)^2}, \; \frac{k_2^2}{(k_2\cdot n)^2}, \; \frac{k_1\cdot k_2}{(k_1\cdot n)(k_2\cdot n)}$$

$$q\cdot k_1 = -\tfrac{1}{2}M_Z^2$$

$$q\cdot k_2 = -\tfrac{1}{2}M_Z^2$$

$$k_1\cdot k_2 = -\tfrac{1}{2}M_Z^2 + m_f^2$$

$$(k_1\cdot\epsilon^\gamma)(k_2\cdot\epsilon^\gamma) = (k_1\cdot k_2)$$

WdecaySoft.prc

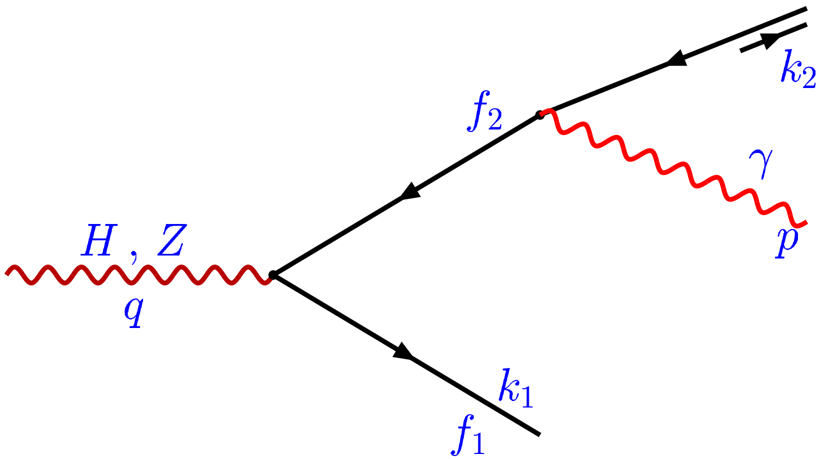
$$\frac{k_1\cdot q}{(k_1\cdot n)M_W}, \; \frac{k_2\cdot q}{(k_2\cdot n)M_W}$$

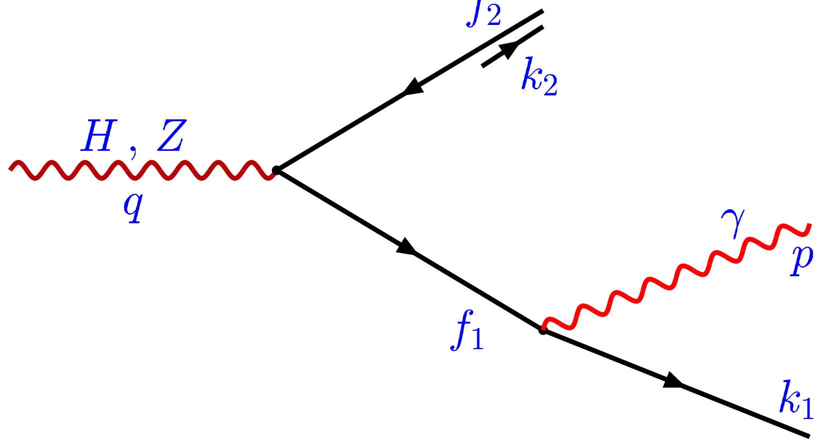
$$q\cdot k_1 = -\tfrac{1}{2}(M_W^2 + m_1^2 - m_2^2)$$

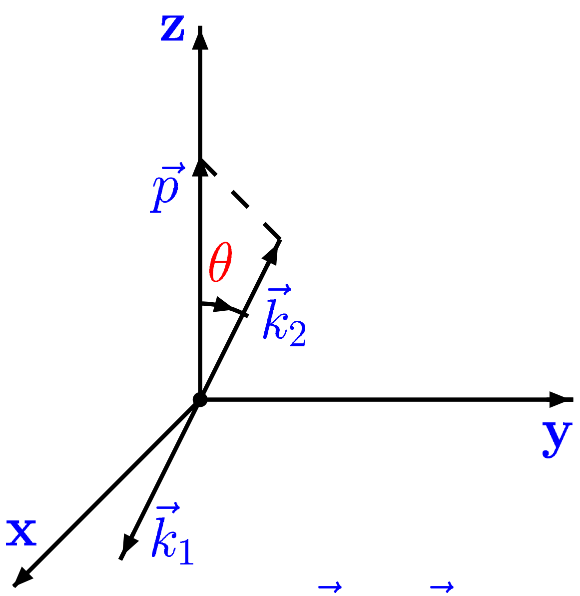
$$q\cdot k_2 = -\tfrac{1}{2}(M_W^2 - m_1^2 + m_2^2)$$

$$k_1\cdot k_2 = -\tfrac{1}{2}(M_W^2 - m_1^2 - m_2^2)$$

5. Hard Bremsstrahlung. 5.1. Neutral bosons decay







$$\vec{k}_1 + \vec{k}_2 = 0$$

Kinematics

$$q^2 = -M_Z^2 \qquad q = k_1 + k_2 + p$$

$$u = -(k_1 + p)^2 \qquad u_- = -\left[(k_1 + p)^2 + m_f^2\right]$$

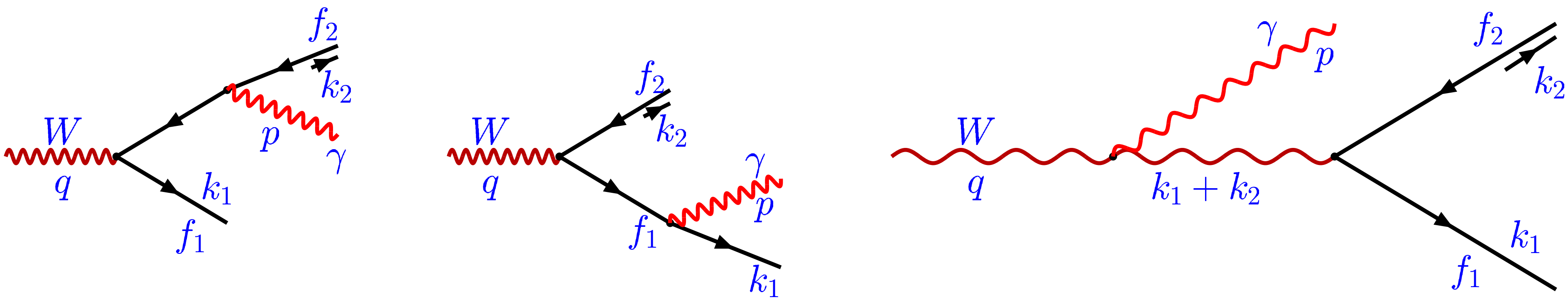
$$t = -(k_2 + p)^2 \qquad t_- = -\left[(k_2 + p)^2 + m_f^2\right]$$

$$s' = -(k_1 + k_2)^2 \qquad u_- + t_- = M_Z^2 - s'$$

Rest system

$$k_1^0 = k_2^0 = \frac{\sqrt{s'}}{2} \qquad p^0 = \frac{M_Z^2 - s'}{2\sqrt{s'}}$$

5. Hard Bremsstrahlung. 5.2. W boson decay



Kinematics

$$u_- = -\left[(k_1+p)^2 + m_1^2\right]$$
$$t_- = -\left[(k_2+p)^2 + m_2^2\right]$$
$$u_- + t_- = M_W^2 - s'$$

$$\lambda' = (s' - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2$$
$$\beta_1' = \frac{|\vec{k}_1|}{k_1^0} = \frac{\sqrt{\lambda'}}{s' + m_1^2 - m_2^2}$$
$$\beta_2' = \frac{|\vec{k}_2|}{k_2^0} = \frac{\sqrt{\lambda'}}{s' - m_1^2 + m_2^2}$$

$f_1 = \nu_e \ , \ f_2 = e^+$

$$\lambda' = (s' - m_e^2)^2$$
$$\beta_1' = \frac{s' - m_e^2}{s' - m_e^2} = 1$$
$$\beta_2' = \frac{s' - m_e^2}{s' + m_e^2}$$

Rest system

$$k_1^0 = \frac{s' + m_1^2 - m_2^2}{2\sqrt{s'}}$$
$$k_2^0 = \frac{s' - m_1^2 + m_2^2}{2\sqrt{s'}}$$
$$p^0 = \frac{M_W^2 - s'}{2\sqrt{s'}}$$

$$\lambda = (M_W^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2$$
$$\beta_1 = \frac{\sqrt{\lambda}}{M_W^2 + m_1^2 - m_2^2}$$
$$\beta_2 = \frac{\sqrt{\lambda}}{M_W^2 - m_1^2 + m_2^2}$$

$$\lambda = (M_W^2 - m_e^2)^2$$
$$\beta_1 = \frac{M_W^2 - m_e^2}{M_W^2 - m_e^2} = 1$$
$$\beta_2 = \frac{M_W^2 - m_e^2}{M_W^2 + m_e^2}$$

Scheme of calculation of **Hard Bremsstrahlung**

BremBff.prc — Bremsstrahlung amplitude M^{Brem} $\nearrow$ **NCdecayHard.prc** $\Longrightarrow$ **NCdecayHardInt.prc**

Table of integrals over $\cos\theta$

Two different Tables of integrals over s'

BffHardSection.prc — $\sum \mid \mathbf{M^{Soft}} \mid^{2}$ $\searrow$ **WdecayHard.prc** — gauge invariance $\Longrightarrow$ **WdecayHardInt.prc**

Integrals

Integrals over $\cos\theta$

$$\int_{-1}^{1} d\cos\theta []$$

$$[] = \frac{1}{u_\_^2}, \ \frac{1}{t_\_^2}, \ \frac{1}{u_\_ t_\_}, \ \frac{1}{u_\_}, \ \frac{1}{t_\_},$$

$$[] = u_\_^{\ 2}, \ t_\_^{\ 2}, \ u_\_ t_\_, \ u_\_, \ t_\_$$

$$t_\_ = 2p^0 k_2^0 (1 - \beta_2' \cos\theta)$$

$$u_\_ = 2p^0 k_1^0 (1 + \beta_1' \cos\theta)$$

Integrals over s'

$$\int_{s'_{min}}^{s'_{max}} ds' [] \qquad s'_{min} = (m_1 + m_2)^2, \quad s'_{max} = M_W^2 - 2M_W \omega$$

$$[] = s'\sqrt{\lambda'}, \ \sqrt{\lambda'}, \ \frac{\sqrt{\lambda'}}{M_W^2 - s'}, \ \frac{\sqrt{\lambda'}}{s'^3}, \ \frac{\sqrt{\lambda'}}{s'^2}, \ \frac{\sqrt{\lambda'}}{s'},$$

$$[] = s' log(\frac{1-\beta_1'}{1+\beta_1'}), \ log(\frac{1-\beta_1'}{1+\beta_1'}), \ \frac{1}{M_W^2 - s'} log(\frac{1-\beta_1'}{1+\beta_1'}),$$

$$[] = s' log(\frac{1-\beta_2'}{1+\beta_2'}), \ log(\frac{1-\beta_2'}{1+\beta_2'}), \ \frac{1}{M_W^2 - s'} log(\frac{1-\beta_2'}{1+\beta_2'})$$

6. Total widths of fermionic boson decays

The Vermaseren's **form3** for analytic calculations realize the following programs:

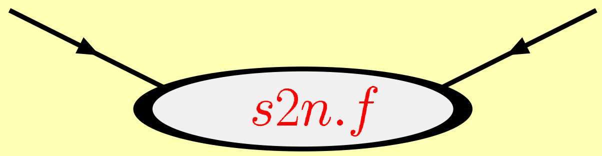
$\underline{NC\_decay\_Brem.frm}$	$\underline{W\_decay\_Brem.frm}$
$call\ NCdecaySoft(typeB, typeU, typeD, 1)$	$call\ WdecaySoft(typeB, typeU, typeD, 1)$
$call\ NCdecayHard(typeB, typeU, typeD, 1)$	$call\ WdecayHard(typeB, typeU, typeD, 1)$
$call\ NCdecayHardInt(typeB, typeU, typeD, 1)$	$call\ WdecayHardInt(typeB, typeU, typeD, 1)$
$call\ NCdecayTotalQED(typeB, typeU, typeD, 1)$	$call\ WdecayTotalQED(typeB, typeU, typeD, 1)$
$call\ NCdecayLimit(typeB, typeU, typeD, 1)$	$call\ WdecayLimit(typeB, typeU, typeD, 1)$

The procedures **NCdecayTotalQED.prc** and **NCdecayTotalQED.prc** put together all contributions to QED radiative corrections of the fermionic widths of boson decays.

$$\Gamma^{Tot} = \Gamma^{Virt} + \Gamma^{Soft} + \Gamma^{Hard}$$

7. Results

$\underline{NC\_decay\_Brem.log}$ $\underline{W\_decay\_Brem.log}$



creates the **fortran** codes