ACAT'2002, 24-28 June, 2002, Moscow, Russia,

About implementation of $e^+e^- \to f\bar{f}$, processes into the framework of `CalcPHEP` system

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AUTOMATIC CALCULATION by CalcPHEP

OF ANY $e^+e^- \to f\bar{f}$ PROCESS

- **IN TWO GAUGES:**
  R_ξ **and Unitary gauge for internal cross checking**
- **ALL $f\bar{f}$ CHANNELS**
- **FULLY MASSIVE CASE AND TWO LIMITS**

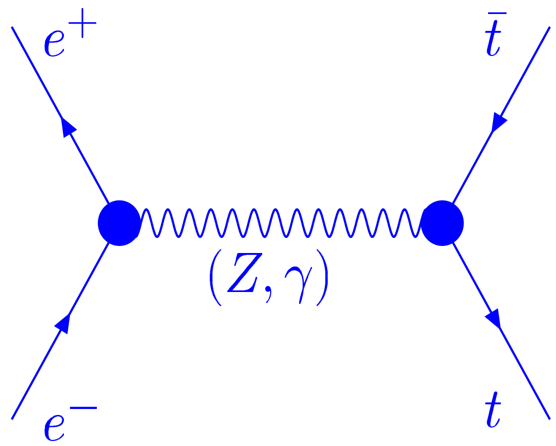
$$\boxed{m_f \longrightarrow 0}$$

$$\boxed{m_{f'} \longrightarrow 0}$$

DONE

P.S. OMS RENORMALIZATION SCHEME
a systematic presentation – the book
D.Bardin, G.Passarino *The Standard Model in the Making*,
Oxford University Press, Oxford, 1999.

AMPLITUDES in L, Q, D - BASIS



$$A \sim \left[i\gamma_\mu \left(1+\gamma_5\right) F_L^e\left(s\right) + i\gamma_\mu F_Q^e\left(s\right)\right] \otimes$$

$$\left[i\gamma_\mu \left(1+\gamma_5\right) F_L^t\left(s\right) + i\gamma_\mu F_Q^t\left(s\right) + m_t I D_\mu F_D^t\left(s\right)\right]$$

$$D_\mu = \left(p_3 - p_4\right)_\mu$$

6 STRUCTURES

Born-like structure of the ONE LOOP AMPLITUDE

in terms of LL, QL, LQ, QQ, LD and QD form factors

$$\mathcal{A}_\gamma = i\frac{4\pi Q_e Q_f}{s}\chi_\gamma(s)\alpha(s)\gamma_\mu \otimes \gamma_\mu$$

$$\mathcal{A}_Z = i\frac{g^2}{16\pi^2}\, e^2\, \frac{\chi_Z(s)}{s}$$

$$\times\Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5)I_e^{(3)}I_t^{(3)}F_{LL}(s,t) + \delta_e I_t^{(3)}\gamma_\mu\otimes\gamma_\mu(1+\gamma_5)F_{QL}(s,t)$$

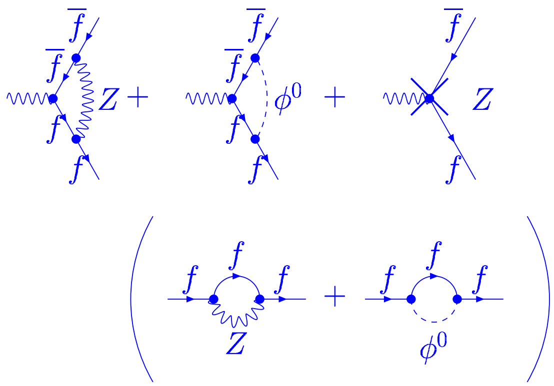
$$+\delta_t I_e^{(3)}\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu F_{LQ}(s,t) + \delta_e\delta_t\gamma_\mu\otimes\gamma_\mu F_{QQ}(s,t)$$

$$+\gamma_\mu(1+\gamma_5)\otimes(-im_tD_\mu)\,I_e^{(3)}I_t^{(3)}F_{LD}(s,t) + \delta_e I_t^{(3)}\gamma_\mu\otimes(-im_tD_\mu)\,F_{QD}(s,t)\Big\}$$

$$\chi_z(s) \;=\; \frac{1}{4s_W^2c_W^2}\frac{s}{s-M_z^2+i\frac{\Gamma_z}{M_z}s} \qquad \delta_f = v_f - a_f = -2Q_f s_W^2$$

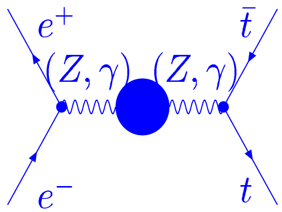
SEVEN GAUGE-INVARIANT SUBSETS of diagrams

- $\{A \longrightarrow \xi_A\}$ – VERTICES + WF REN A CLUSTER
- $\{A \longrightarrow \xi_A\}$ – AA BOX
- $\{A, Z \longrightarrow \xi_A \xi_Z\}$ – ZA BOX
- $\{Z \longrightarrow \xi_Z\}$ – ZZ BOX
- $\{Z, \phi^0 \longrightarrow \xi_Z\}$ Z CLUSTER $\Longrightarrow$

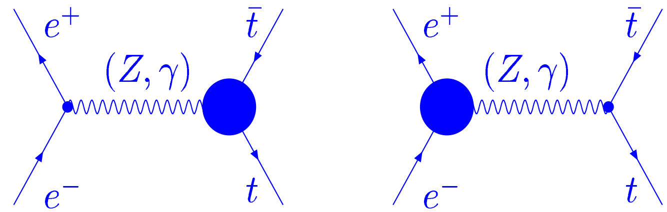


- $\{H, \phi^0 \longrightarrow \xi_Z\}$ H CLUSTER
- $\{W, \phi^\pm \longrightarrow \xi\}$ W CLUSTER
- + self-energies and WW box

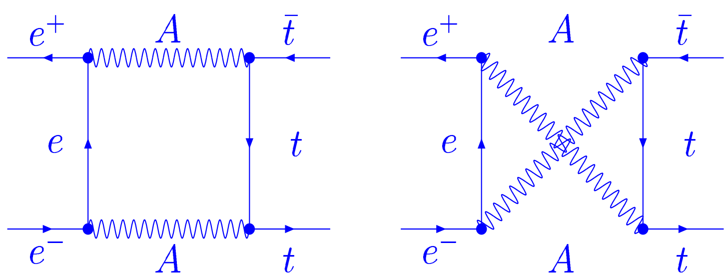
BUILDING BLOCKS



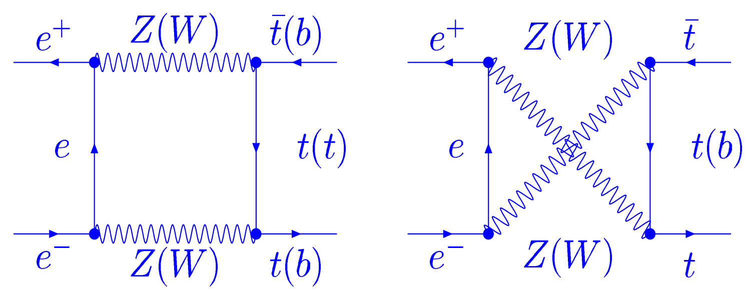
Self-Energies



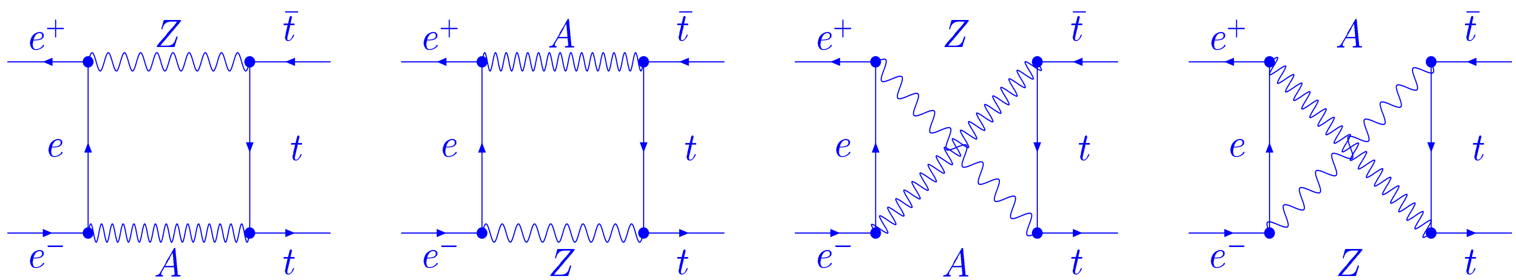
Vertices



Direct and crossed AA boxes

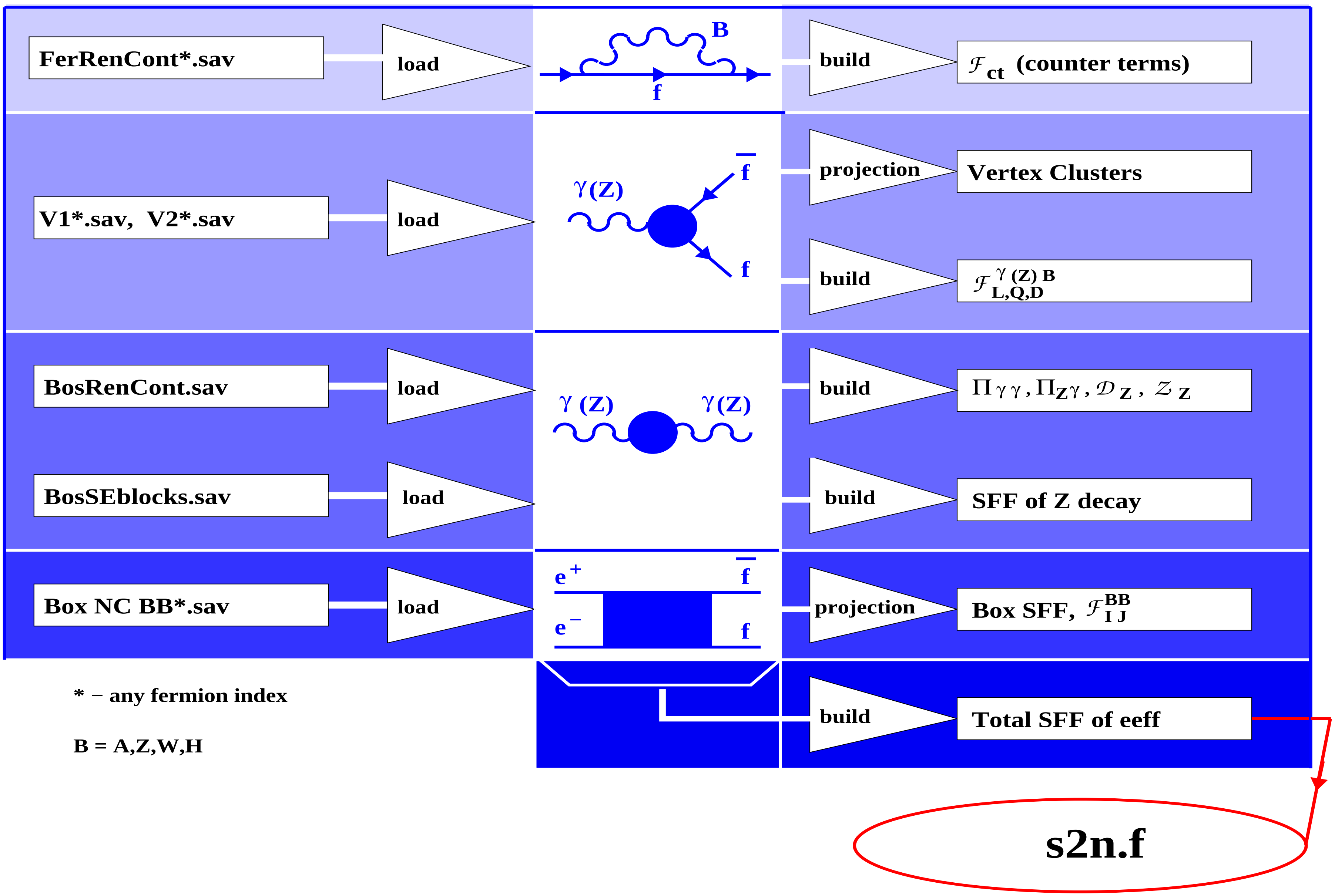


Direct and crossed $ZZ(WW)$ boxes



Direct and crossed ZA boxes

Calculation flow in eeffRen.frm



Total SCALAR FORM FACTORS (SFF)

The ultraviolet-finite results for six SFFs are:

$$
\begin{aligned}
F_{LL}(s,t,u) &= \mathcal{F}_L^{zee}(s) + \mathcal{F}_L^{ztt}(s) + \mathcal{F}_{LL}^{ct}(s) + \mathcal{F}_{LL}^{\rm BOX}(s,t,u) \\
F_{QL}(s,t,u) &= \mathcal{F}_Q^{zee}(s) + \mathcal{F}_L^{ztt}(s) + k\;\mathcal{F}_L^{\gamma tt}(s) + \mathcal{F}_{QL}^{ct}(s) + \mathcal{F}_{QL}^{\rm BOX}(s,t,u) \\
F_{LQ}(s,t,u) &= \mathcal{F}_L^{zee}(s) + \mathcal{F}_Q^{ztt}(s) + k\;\mathcal{F}_L^{\gamma ee}(s) + \mathcal{F}_{LQ}^{ct}(s) + \mathcal{F}_{LQ}^{\rm BOX}(s,t,u) \\
F_{QQ}(s,t,u) &= \mathcal{F}_Q^{zee}(s) + \mathcal{F}_Q^{ztt}(s) - \frac{k}{s_W^2}\left[\mathcal{F}_Q^{\gamma ee}(s) + \mathcal{F}_Q^{\gamma tt}(s)\right] + \mathcal{F}_{QQ}^{ct}(s) + \mathcal{F}_{QQ}^{\rm BOX}(s,t,u) \\
F_{LD}(s,t,u) &= \mathcal{F}_D^{ztt}(s) + \mathcal{F}_{LD}^{\rm BOX}(s,t,u) \\
F_{QD}(s,t,u) &= \mathcal{F}_D^{ztt}(s) + k\;\mathcal{F}_D^{\gamma tt}(s) + \mathcal{F}_{QD}^{\rm BOX}(s,t,u)
\end{aligned}
$$

$$
k = c_W^2\left(M_Z^2/s - 1\right)
$$

For the IJ component of a box contribution we have:

$$
\mathcal{F}_{IJ}^{\rm BOX}(s,t,u) = k^{AA}\mathcal{F}_{IJ}^{AA}(s,t,u) + k^{ZA}\mathcal{F}_{IJ}^{ZA}(s,t,u) + k^{ZZ}\mathcal{F}_{IJ}^{ZZ}(s,t,u) + k^{WW}\mathcal{F}_{IJ}^{WW}(s,t,u)
$$

Moreover,

$$
\mathcal{F}_{L,Q,D}^{\gamma(z)tt}(s) = \sum_{B=A,Z,H,W} \mathcal{F}_{L,Q,D}^{\gamma(z)B}(s)
$$

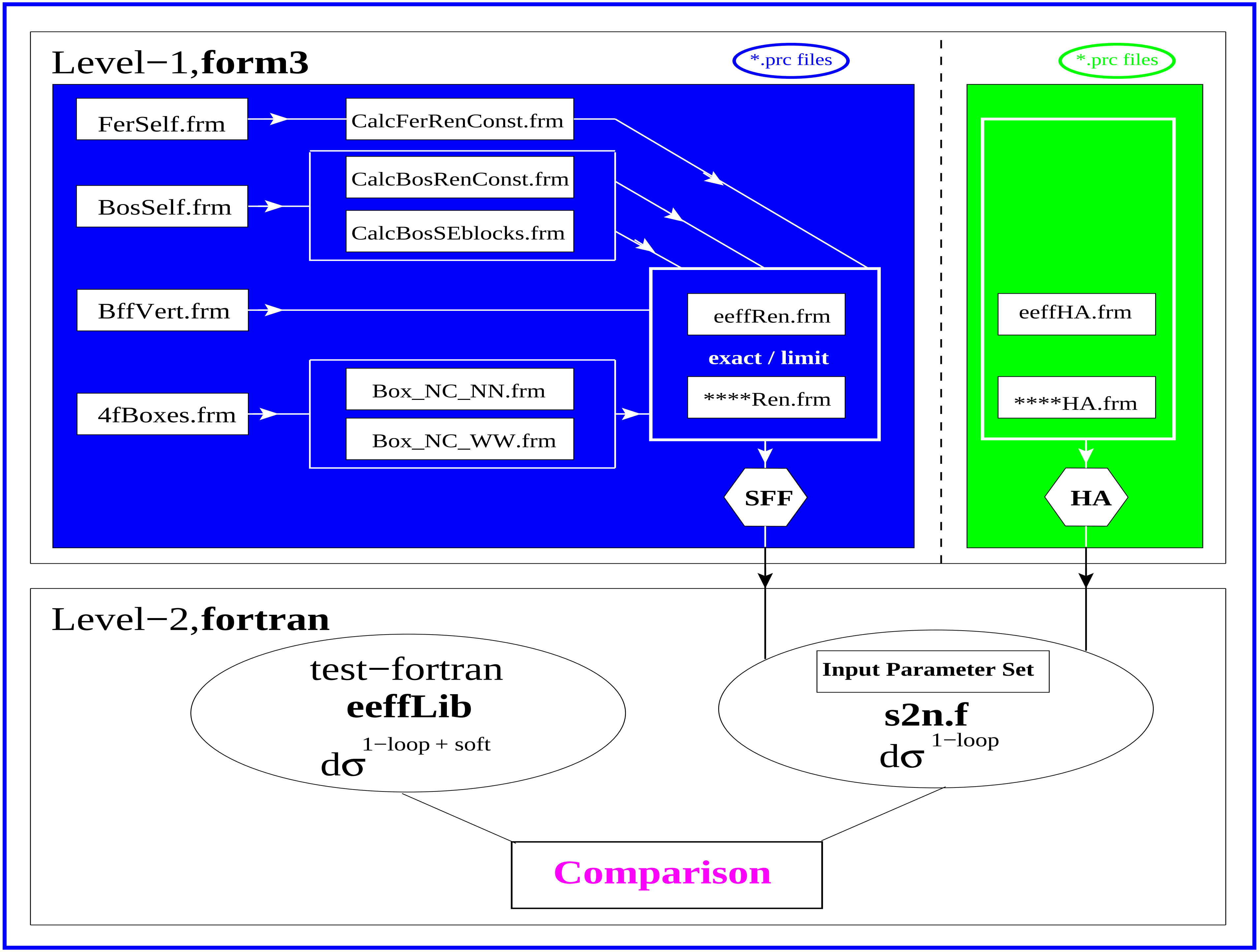
except for $\mathcal{F}_L^{\gamma A}(s) = 0$ and $\mathcal{F}_L^{\gamma H}(s) = 0$.

$e^+e^- \to t\bar{t}$ process in the HELICITY AMPLITUDES (HA)

16 Helicity Amplitudes for any $2f \to 2f$ process.
The unpolarized case, the electron mass is ignored $\longrightarrow$ **6 HAs**,
which depend on kinematical variables, coupling constants and 6 SFFs:

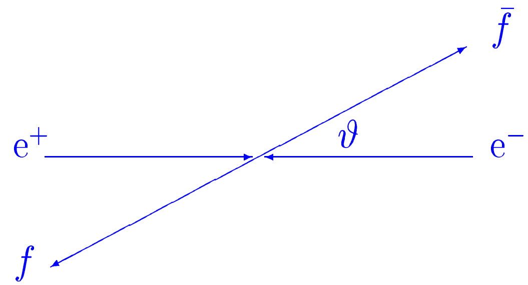
$$
\begin{aligned}
\mathcal{A}_{++++} &= 0, \qquad \mathcal{A}_{+++-} = 0, \qquad \mathcal{A}_{++-+} = 0, \qquad \mathcal{A}_{++--} = 0,\\
\mathcal{A}_{+-+-} &= s\,(1-\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\delta_e\left[\,(1+\beta_t)\,I_t^{(3)}F_{QL} + \delta_tF_{QQ}\right]\right),\\
\mathcal{A}_{+--+} &= s\,(1+\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\delta_e\left[(1-\beta_t)I_t^{(3)}F_{QL} + \delta_tF_{QQ}\right]\right),\\
\mathcal{A}_{+---} &=\\
\mathcal{A}_{+-++} &= 2\sqrt{s}m_t\sin\vartheta\left(Q_eQ_tF_{GG} + \chi_Z\delta_e\left[I_t^{(3)}F_{QL} + \delta_tF_{QQ} + \frac{1}{2}s{\beta_t}^2I_t^{(3)}F_{QD}\right]\right),\\
\mathcal{A}_{-+++} &=\\
\mathcal{A}_{-+--} &= -2\sqrt{s}m_t\sin\vartheta\left(Q_eQ_tF_{GG} + \chi_Z\left[2I_e^{(3)}I_t^{(3)}F_{LL} + 2I_e^{(3)}\delta_tF_{LQ} + \delta_eI_t^{(3)}F_{QL} + \delta_e\delta_tF_{QQ} + \frac{1}{2}s{\beta_t}^2I_t^{(3)}\left(2I_e^{(3)}F_{LD} + \delta_eF_{QD}\right)\right]\right),\\
\mathcal{A}_{-++-} &= s\,(1+\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\left[\,(1+\beta_t)\left(2I_e^{(3)}I_t^{(3)}F_{LL} + \delta_eI_t^{(3)}F_{QL}\right) + \delta_t\left(2I_e^{(3)}F_{LQ} + \delta_eF_{QQ}\right)\right]\right),\\
\mathcal{A}_{-+-+} &= s\,(1-\cos\vartheta)\left(Q_eQ_tF_{GG} + \chi_Z\left[\,(1-\beta_t)\,I_t^{(3)}\left(2I_e^{(3)}F_{LL} + \delta_eF_{QL}\right) + \delta_t\left(2I_e^{(3)}F_{LQ} + \delta_eF_{QQ}\right)\right]\right),\\
\mathcal{A}_{--++} &= 0, \qquad \mathcal{A}_{--+-} = 0, \qquad \mathcal{A}_{---+} = 0, \qquad \mathcal{A}_{----} = 0.
\end{aligned}
$$

$$\cos\vartheta = \left(t - m_t^2 + s/2\right)2/(s\beta_t)\,, \quad \beta_t^2 = 1 - 4m_t^2/s, \quad \delta_f = v_f - a_f.$$



Level−1,form3
*.prc files
*.prc files
FerSelf.frm
CalcFerRenConst.frm
BosSelf.frm
CalcBosRenConst.frm
CalcBosSEblocks.frm
BffVert.frm
eeffRen.frm
exact / limit
****Ren.frm
4fBoxes.frm
Box_NC_NN.frm
Box_NC_WW.frm
eeffHA.frm
****HA.frm
SFF
HA
Level−2,fortran
test−fortran
eeffLib
dσ 1−loop + soft
Input Parameter Set
s2n.f
dσ 1−loop
Comparison

DIFFERENTIAL CROSS SECTION $\left(\frac{d\sigma^{\text{virt+soft}}}{d\cos\vartheta}\right)^{eeff}$

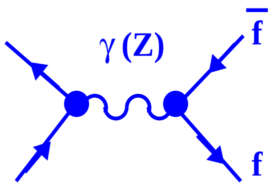


For the unpolarized case:

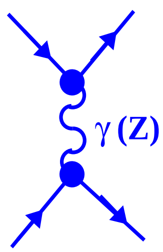
$$\frac{d\sigma}{d\cos\vartheta} = \frac{\pi\alpha^2}{s^3}\beta_t N_c \sum_{\lambda_i\lambda_j\lambda_k\lambda_l} \left|\mathcal{A}_{\lambda_i\lambda_j\lambda_k\lambda_l}\right|^2$$

For the amplitude $\mathcal{A}_{\lambda_i\lambda_j\lambda_k\lambda_l}$ each index $\lambda_{(i,j,k,l)}$ takes two values ($\pm = \pm 1$) meaning 2 times the projection of spins $e^+, e^-, t, \bar{t}$ onto their corresponding momentum.

DONE

I) 

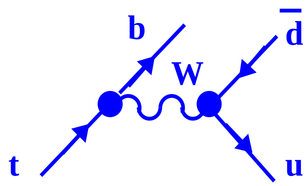
e e --> $\nu\bar{\nu}$, l l, u $\bar{u}$, d $\bar{d}$, c $\bar{c}$, s $\bar{s}$, b $\bar{b}$, t $\bar{t}$ (LEVEL 1−2)

II) 

ANY NC t−channel (LEVEL 1)

NOT DONE

I) **s−, t−channel processes and 1 --> 3 decay mediated by CC,**

for instance, 

(however, basement is ready).

COMPARISON eeffLib–ZFITTER

- For the SFFs **AGREEMENT** within **8-9** digits
- For the complete 1-loop differential cross section $d\sigma^{(1)}/d\cos\vartheta$ **AGREEMENT** within **7-8** digits
- For the total 1-loop cross section and $\sigma_{\rm FB}$ **AGREEMENT** for the light fermion masses within **6-7** digits

first row – ZFITTER second row – eeffLib.

channel	100 GeV		200 GeV		300 GeV	
	$\sigma_{\rm tot}$	$\sigma_{\rm FB}$	$\sigma_{\rm tot}$	$\sigma_{\rm FB}$	$\sigma_{\rm tot}$	$\sigma_{\rm FB}$
$\nu\bar{\nu}$, $m_\nu = 0$	84.81710 84.81710	9.509864 9.509865	0.963362 0.963362	0.089665 0.089665	0.320985 0.320985	0.021592 0.021592
e^-e^+, $m_e = 0$	52.61662 52.61662	30.78899 30.78899	2.980668 2.980667	1.654673 1.654673	1.276008 1.276008	0.648414 0.648414
$\mu^+\mu^-$, $m_\mu = 0.106$ GeV	52.61634 52.61634	30.78885 30.78885	2.980667 2.980667	1.654671 1.654671	1.276008 1.276008	0.648414 0.648414
$\tau^+\tau^-$, $m_\tau = 1.77705$ GeV	52.53632 52.53661	30.75010 30.75024	2.980435 2.980443	1.654149 1.654156	1.275972 1.275974	0.648324 0.648326
$u\bar{u}$, $m_u = 0.1$ GeV	160.8980 160.8981	70.98406 70.98416	5.021808 5.021810	3.360848 3.360848	2.031754 2.031754	1.269556 1.269556
$d\bar{d}$, $m_d = 0.1$ GeV	193.7658 193.7658	50.03208 50.03227	3.120724 3.120725	1.867871 1.867871	1.149479 1.149479	0.725581 0.725581
$b\bar{b}$, $m_b = 0.1$ GeV	191.0416 191.0416	49.76543 49.76562	3.134547 3.134547	1.892628 1.892629	1.149243 1.149243	0.720476 0.720476
$b\bar{b}$, $m_b = 4.7$ GeV	189.6321 189.3855	49.39098 49.32791	3.129824 3.129858	1.888530 1.888542	1.148541 1.148565	0.719805 0.719802

COMPARISON with a code generated by `s2n.f`

- For the complete 1-loop differential cross-sections $d\sigma^{(1)}/d\cos\vartheta$, for a standard input parameter set ($m_t = 173.8$ GeV)
  — **AGREEMENT** within **12** or **13** digits

first row `eeffLib` second row `s2n.f`

$\sqrt{s}$	400 GeV	700 GeV	1000 GeV
$\cos\vartheta$			
–0.9	0.22357662754774	0.06610825350063	0.02926006442715
	0.22357662754769	0.06610825350063	0.02926006442715
0.0	0.34494634728716	0.14342802645636	0.06752160108814
	0.34494634728707	0.14342802645634	0.06752160108813
0.9	0.54806778978208	0.33837133344667	0.16973989931024
	0.54806778978194	0.33837133344664	0.16973989931023

COMPARISON with the other codes

- For the 1-loop cross-section **without soft**
  — **AGREEMENT 11** digits with FeynArts

- For the 1-loop cross-section, **with soft**
  — **AGREEMENT** within 8 digits with Bielefeld–Zeuthen group
  (hep-ph/0202102, J. Fleisher et al.)

$\frac{d\sigma^{(1)}}{d\cos\vartheta}$ for the process $e^+e^- \to t\bar{t}$ with soft photons, $E_\gamma^{\max} = \sqrt{s}/10$.

$\sqrt{s}$	400 GeV	700 GeV	1000 GeV
$\cos\vartheta$			
–0.9	0.17613018248935	0.05199100267864	0.02310170508071
–0.5	0.21014509428358	0.06560630503586	0.02882301902010
0.0	0.27268108572063	0.11496514450150	0.05495088904853
0.5	0.35592722356682	0.19615154401629	0.09941700898317
0.9	0.43637377538440	0.27915043976042	0.14426233253975