ACAT'2002, 24-28 June, 2002, Moscow, Russia

A Monte Carlo simulation of decays within the CalcPHEP project

The library of Monte Carlo programs for simulation of two particle leptonic and quark decays of the W,Z and H(Higgs) bosons with single bremsstrahlung photon emission as a part of CalcPHEP system is described. The QED and EW $O(\alpha)$ radiative corrections are implemented without any approximation, keeping the masses of decay product particles. The decay amplitudes are evaluated numerically using Kleiss&Stirling helicity amplitude method. The comparison with PHOTOS — A universal Monte Carlo simulator for QED radiative corrections is also presented.

G. Nanava.

Laboratory of Nuclear Problems, joint institute for Nuclear Research

A Brief Description of the Monte Carlo Techniques

We use the Monte Carlo methods for the computation of decay partial widths of the SM gauge bosons (H,Z and W) and various differential distributions for these decays. In our case the decay width is the five dimensional integral over the allowed phase space:

$$\Gamma = \frac{1}{2\,M} \int X(p_1, p_2, k)\, d\Phi_3 \qquad (1)$$

where, $X(p_1, p_2, k)$ denotes the matrix element squared, summed and averaged over spins and colors, and $d\Phi_3$ is the 3-particle phase space volume. The amplitude calculation methods and the phase space parametrization are given below. By additional change of variables the phase space volume can be expressed in terms of variables that span unit hyper cube i.e. all the five variables running over an interval (0,1) and as a result we obtain expression for (1) suitable for Monte Carlo computation.

$$\Gamma = \int W(r_1, r_2, r_3, r_4, r_5)\, dr_1 dr_2 dr_3 dr_4 dr_5 \qquad (2)$$

where $W(r_1, r_2, r_3, r_4, r_5) = X(p_1, p_2, k) J$, J is Jacobian of this transformation. The Monte Carlo recipe for event generation is based on the following: in the first step the four–momenta p_1, p_2, k (the event) are calculated in the rest frame of the decaying particle out of uniformly distributed random numbers r_i, then the corresponding weight $W(r_1, r_2, r_3, r_4, r_5)$ is calculated. For production of unweighted events we use the rejection method as follows: for each event we compute rejection weight $w = W/W_{max}$ and compare it with random number r, where W_{max} is a maximum weight and r $\in$(0,1), if $w < r$ we accept the event, else we reject it. Accepted events will be distributed according to $X(p_1, p_2, k)$ and as a byproduct the integral i.e. decay width Γ is given by:

$$\Gamma = < W > \text{, with a variance} \quad \sigma = \left\langle < W^2 > - < W >^2 \right\rangle \qquad (3)$$

where the average is taken over all generated events.

Feynman Diagram Calculation With Kleiss&Stirling Spin-Amplitude Method

Motivation

The calculation of cross sections and decay widths for the production of many particles(leptons, photons, quarks, gluons,..) has traditionally been restricted by the technical difficulties associated with the evaluation of the corresponding multiparticle Feynman diagrams. For processes with more than two particle in the final state the spin-projection operator methods quickly become unwieldy. This difficulty is overcome by employing the spinor helicity method, which is powerful technique for computing helicity amplitudes for multiparticle processes. For evaluation decay amplitudes we used Kleiss&Stirling spinor method. Typically, this spinor technique have given access to one or two higher orders in complexity compared to the usual trace techniques for squares of amplitudes.

A brief description of Kleiss&Stirling spinor method

The basic idea is to reduce the amplitudes to an expression involving only spinor products (objects like $\bar{u}_{\lambda_1}(p_1)u_{\lambda_2}(p_2)$, see below), which itself can be expressed in terms of the four vectors involving in the precesses. The arbitrary massless spinor $u_\lambda(p)(p^2=0)$ can be constructed out of the two basic spinors $\mathbf{u}_\lambda(\zeta)$, which represent the helicity states of a massless fermion of momentum ζ. They have the following properties:

* Dirac equation

$$\widehat{\zeta}\,\mathbf{u}_\lambda(\zeta) = 0, \qquad \widehat{\zeta} = \zeta^\mu\gamma_\mu$$
$$\omega_\lambda\mathbf{u}_\lambda(\zeta) = \mathbf{u}_\lambda(\zeta),\ \omega_\lambda = \frac{1}{2}(1+\lambda\gamma_5)$$

* Normalization and Completeness

$$\bar{\mathbf{u}}_\lambda(\zeta)\,\gamma^\mu\,\mathbf{u}_\lambda(\zeta) = 2\,\zeta^\mu$$
$$\mathbf{u}_\lambda(\zeta)\,\bar{\mathbf{u}}_\lambda(\zeta) = \widehat{\zeta}\,\omega_\lambda \qquad (3)$$

The relative phase of $\mathbf{u}_+(\zeta)$ and $\mathbf{u}_-(\zeta)$ is fixed by choosing the arbitrary four momentum η^μ with the following properties :

$$\begin{aligned} \mathbf{u}_\lambda(\zeta) &= \lambda\,\widehat{\eta}\,\mathbf{u}_{-\lambda}(\zeta) \\ \eta.\,\zeta &= 0, \qquad \eta^2 = -1, \qquad \zeta^2 = 0. \end{aligned} \tag{4}$$

Any other ***massless*** spinor may be defined as:

$$u_\lambda(p) = \frac{1}{\sqrt{2p.\,\zeta}}\,\widehat{p}\;\mathbf{u}_{-\lambda}(\zeta)\,, \qquad v_\lambda(p) = \frac{1}{\sqrt{2p.\,\zeta}}\,\widehat{p}\;\mathbf{u}_{\lambda}(\zeta) \tag{5}$$

using (1)-(2) one can easily check that the usual relations (Dirac equation, Completeness and Normalization conditions) for $u_\lambda(p)$ and $u_\lambda(p)$ spinors hold.
Spinors for the ***massive*** particle with four-momentum p and mass m can also be defined in term of basic massless spinors:

$$\begin{aligned} u_\lambda(p) &= \frac{\widehat{p}+m}{\sqrt{2p.\,\zeta}}\mathbf{u}_{-\lambda}(\zeta)\,, \qquad v_\lambda(p) = \frac{\widehat{p}-m}{\sqrt{2p.\,\zeta}}\mathbf{u}_{\lambda}(\zeta) \\ \bar{u}_\lambda(p) &= \bar{\mathbf{u}}_{-\lambda}(\zeta)\frac{\widehat{p}+m}{\sqrt{2p.\,\zeta}}\,, \qquad \bar{v}_\lambda(p) = \bar{\mathbf{u}}_{\lambda}(\zeta)\frac{\widehat{p}-m}{\sqrt{2p.\,\zeta}} \end{aligned} \tag{6}$$

These spinors obey the following relations:

$$\begin{aligned} \widehat{p}\,u_\lambda(p) &= m u_\lambda(p)\,, \qquad & u_\lambda(p)\,\bar{u}_\lambda(p) &= \frac{1+\lambda\,\gamma_5\,\widehat{s}}{2}\,(\widehat{p}+m) \\ \widehat{p}\,v_\lambda(p) &= -m\,v_\lambda(p)\,, \qquad & v_\lambda(p)\,\bar{v}_\lambda(p) &= \frac{1+\lambda\,\gamma_5\,\widehat{s}}{2}\,(\widehat{p}-m) \end{aligned} \tag{7}$$

where

$$s^\mu = \frac{p^\mu}{m} - \frac{m}{p.\,\zeta}\,\zeta^\mu, \qquad s^2 = -1, \qquad s.\,p = 0 \tag{8}$$

is the spin quantization vector in the fermion rest frame, it points opposite to ζ, i.e. $\vec{s} \sim -\vec{\zeta}$, thus the spin quantization axis for every fermion is guided by single and common massless vector ζ.

For non-zero ***spinor*** (***inner***) ***products*** for ***massless*** fermion we have :

$$
\begin{aligned}
s_+(p_1,p_2) &\equiv \bar{u}_+(p_1)\,u_-(p_2), \qquad s_\lambda(p_1,p_2) = -s_\lambda(p_2,p_1) \\
s_-(p_1,p_2) &\equiv \bar{u}_-(p_1)\,u_+(p_2), \qquad s_\lambda(p_1,p_2) = -s_{-\lambda}(p_2,p_1)^*, \quad |s_\lambda(p_1,p_2)|^2 = 2p_1.\,p_2
\end{aligned}
\tag{9}
$$

Using (1)-(4) one can easily find expression for $s_\lambda(p_1,p_2)$ in any reference frame in terms of p_1 and p_2, which is suitable for numerical evaluation:

$$
\begin{aligned}
s_+(p_1,p_2) &= 2(2p_1.\,\zeta)^{-1/2}(2p_2.\,\zeta)^{-1/2}\left[(p_1.\,\zeta)(p_2.\,\eta)-(p_1.\,\eta)(p_2.\,\zeta)-i\epsilon_{\mu\nu\rho\sigma}\zeta^\mu\eta^\nu p_1^\rho p_2^\sigma\right] \\
& s_-(p_1,p_2) = -s_+(p_1,p_2)^*
\end{aligned}
\tag{10}
$$

In actual computations we can specify ζ and η so that the $s_\lambda(p_1,p_2)$ becomes quite compact. In our calculations we use $\zeta = (1,1,0,0)$ and $\eta = (0,0,1,0)$, which leads to the following ***massless inner product***:

$$
s_+(p_1,p_2) = -(p_2^y + i\,p_2^z)\sqrt{\frac{p_1^0-p_1^x}{p_2^0-p_2^x}} + (p_1^y + i\,p_1^z)\sqrt{\frac{p_2^0-p_2^x}{p_1^0-p_1^x}} \tag{11}
$$

Similarly, inner product for massive fermion comes from (5):

$$
\begin{aligned}
\bar{u}_{\lambda_1}(p_1)\,u_{\lambda_2}(p_2) &\equiv S(p_1,m_1,\lambda_1,p_2,m_2,\lambda_2), \\
\bar{u}_{\lambda_1}(p_1)\,v_{\lambda_2}(p_2) &= S(p_1,m_1,\lambda_1,p_2,-m_2,-\lambda_2), \\
\bar{v}_{\lambda_1}(p_1)\,u_{\lambda_2}(p_2) &= S(p_1,-m_1,-\lambda_1,p_2,m_2,\lambda_2), \\
\bar{v}_{\lambda_1}(p_1)\,v_{\lambda_2}(p_2) &= S(p_1,-m_1,-\lambda_1,p_2,-m_2,-\lambda_2),
\end{aligned}
\tag{12}
$$

where

$$
S(p_1,m_1,\lambda_1,p_2,m_2,\lambda_2) = \delta_{\lambda_1,-\lambda_2}\,s_{\lambda_1}(p_{1\zeta},p_{2\zeta}) + \delta_{\lambda_1,\lambda_2}\left(m_1\sqrt{\frac{2\,\zeta\,p_2}{2\,\zeta\,p_1}} + m_2\sqrt{\frac{2\,\zeta\,p_1}{2\,\zeta\,p_2}}\right) \tag{13}
$$

In order to use spinor techniques described above for processes with external massless or massive spin-1 particles (photon, Z,W-bosons, ..) one have to find an expression of the polarization vector in terms of spinor inner product.

Polarization Vectors of Massless Bosons:

For photons with momentum k^μ and helicity $\sigma = \pm$ we use Kleiss&Stirling definition of polarization vectors in the axial gauge:

$$\varepsilon^\mu_\sigma(k,\beta) = \frac{\bar{u}_\sigma(k)\gamma^\mu u_\sigma(\beta)}{\sqrt{2}s_{-\sigma}(k,\beta)}, \qquad \varepsilon^\mu_\sigma(k,\zeta) = \frac{\bar{u}_\sigma(k)\gamma^\mu \mathbf{u}_\sigma(\zeta)}{\sqrt{2}s_{-\sigma}(k,\zeta)} \tag{12}$$

where β is any light-like four-vector not collinear to k^μ or ζ^μ. The vectors $\varepsilon^\mu_\sigma(k,\beta)$ obey usual relations:

$$\begin{aligned} \varepsilon_{\sigma_1}(k,\beta).\,\varepsilon^*_{\sigma_2}(k,\beta) &= -\delta_{\sigma_1\sigma_2}, \qquad \varepsilon_\sigma(k,\beta).\,k = 0, \qquad \varepsilon^\mu_\sigma(k,\beta)^* = \varepsilon^\mu_{-\sigma}(k,\beta) \\ \sum_\sigma \varepsilon^\mu_\sigma(k,\beta)\,\varepsilon^\nu_\sigma(k,\beta)^* &= -g^{\mu\nu} + \frac{k^\mu\beta^\nu + k^\nu\beta^\mu}{k.\,\beta} \end{aligned} \tag{13}$$

Using the Chisholm identity $\bar{u}_\sigma(k)\gamma^\mu u_\sigma(\beta) = 2u_\sigma(\beta)\bar{u}_\sigma(k) + 2u_{-\sigma}(k)\bar{u}_{-\sigma}(\beta)$ we get more useful expressions:

$$\varepsilon^\mu_\sigma(k,\beta) = \frac{\sqrt{2}\,[u_\sigma(\beta)\bar{u}_\sigma(k) + u_{-\sigma}(k)\bar{u}_{-\sigma}(\beta)]}{s_{-\sigma}(k,\beta)} \tag{14}$$

Polarization Vectors of Massive Bosons:

Polarization vectors for boson with momentum k, mass m and helicity $\sigma = 0, \pm$ can be expressed in terms of two light-like four-vectors:

$$\begin{aligned} \varepsilon^\mu_\sigma(k) &= \frac{\bar{u}_\sigma(k_1)\gamma^\mu u_\sigma(k_2)}{\sqrt{2}\,m}, \qquad \sigma = \pm \\ \varepsilon^\mu_0(k) &= \frac{\bar{u}_+(k_1)\gamma^\mu u_+(k_1) - \bar{u}_+(k_2)\gamma^\mu u_+(k_2)}{2\,m} = \frac{k^\mu_1 - k^\mu_2}{m} \end{aligned} \tag{15}$$

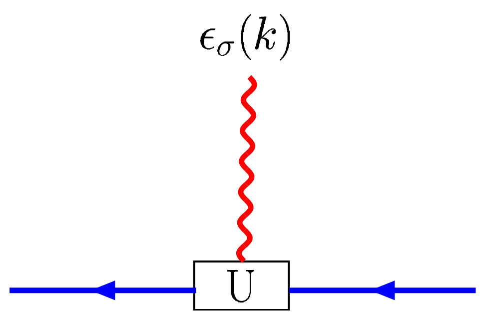
where

$$ k_1{}^\mu = \frac{k^\mu - m\, n^\mu}{2}, \qquad k_2{}^\mu = \frac{k^\mu + m\, n^\mu}{2}, \qquad k_1{}^2 = k_2{}^2 = 0 $$

and n^μ is spin vector: $n.\,k = 0, \qquad n^2 = -1$

All the properties of massive boson polarization vector hold and they are the same as (15) except completeness relation, which has the form: $\sum_\sigma \varepsilon^\mu_\sigma(k)\, \varepsilon^\nu_\sigma(k)^* = -g^{\mu\nu} + k^\mu\, k^\nu / m^2$.

For computation of amplitudes with real boson emission (massive or massless) it is useful to use the following building block (transition matrix):



$$
\begin{aligned}
\bar{u}_{\lambda_1}(p_1)\, \widehat{\epsilon}_\sigma(k)\, (v_f + a_f \gamma_5)\, u_{\lambda_2}(p_2) &\equiv U^\sigma_{\lambda_1,\lambda_2}(k, p_1, m_1, p_2, m_2) \\
\bar{v}_{\lambda_1}(p_1)\, \widehat{\epsilon}_\sigma(k)\, (v_f + a_f \gamma_5)\, u_{\lambda_2}(p_2) &= U^\sigma_{-\lambda_1,\lambda_2}(k, p_1, -m_1, p_2, m_2) \\
\bar{u}_{\lambda_1}(p_1)\, \widehat{\epsilon}_\sigma(k)\, (v_f + a_f \gamma_5)\, v_{\lambda_2}(p_2) &= U^\sigma_{\lambda_1,-\lambda_2}(k, p_1, m_1, p_2, -m_2) \\
\bar{v}_{\lambda_1}(p_1)\, \widehat{\epsilon}_\sigma(k)\, (v_f + a_f \gamma_5)\, v_{\lambda_2}(p_2) &= U^\sigma_{-\lambda_1,-\lambda_2}(k, p_1, -m_1, p_2, -m_2)
\end{aligned} \tag{14}
$$

where v_f and $a_f$denote the vector and axial coupling constant. $\sigma = 0, \pm$ or $\pm$ for a massive or massless boson. For example for photon case: $v_f = 1$, $a_f = 0$ and $\sigma = \pm$; for $W^\pm$: $v_f = 1$, $a_f = 1$ and $\sigma = 0, \pm$;

The above spinor techniques we use for computation H,Z and W decay amplitudes with one real photon emission.

$H \to f\bar{f}\gamma$

The gauge invariant group of Feynman diagrams for $H \to f\bar{f}\gamma$ decay are depicted in Fig.1.

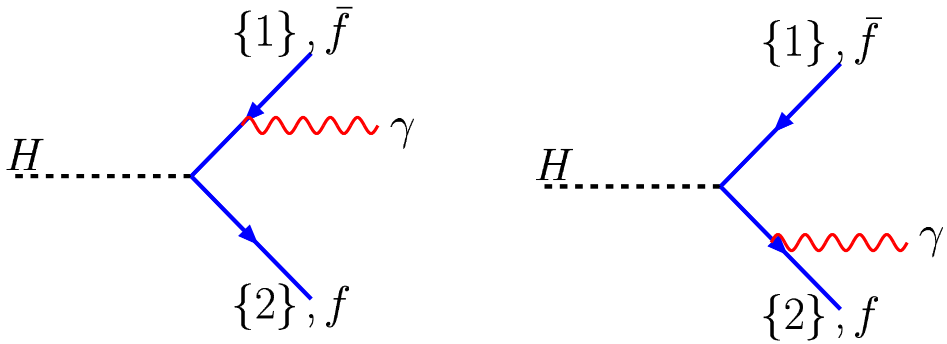


Fig. 1: Feynman diagrams for the decay $H \to f\bar{f}\gamma$

where f denotes leptons($e^{\pm}$,$\mu^{\pm}$,$\tau^{\pm}$) or quarks (u,$\bar{u}$,d, etc.), $\{1\} \equiv (p_1,m_1,\lambda_1)$ and $\{2\} \equiv (p_2,m_2,\lambda_2)$ The helicity amplitudes for this decay have the form:

$$
\begin{aligned}
M^{\sigma}_{\lambda_1,\lambda_2}(k,p_1,p_2) \;=\; & \left(\frac{eQ_f}{2\,k.\,p_2} b_\sigma(k,p_2) - \frac{eQ_f}{2\,k.\,p_1} b_\sigma(k,p_1)\right) B_{\lambda_2,\lambda_1}(p_2,p_1) \\
& + \frac{eQ_f}{2\,k.\,p_2} \sum_{\rho=\pm} U^{\sigma}_{\lambda_2,\rho}(p_2,m_2,k,0,k,0)\, B_{\rho,-\lambda_1}(k,p_1) \\
& - \frac{eQ_f}{2\,k.\,p_1} \sum_{\rho=\pm} B_{\lambda_2,-\rho}(p_2,k)\, U^{\sigma}_{-\rho,-\lambda_1}(k,0,k,0,p_1,m_1)
\end{aligned} \tag{15}
$$

where $B_{\lambda_2,\lambda_1}(p_2,p_1) = -\dfrac{g\,m_f}{2m_w} S(p_2,m_2,\lambda_2,p_1,-m_1,-\lambda_1)$ is the Born-like amplitude.

$Z \to f\bar{f}\gamma$

The Feynman diagrams for $Z \to f\bar{f}\gamma$ decay are shown in Fig.2.

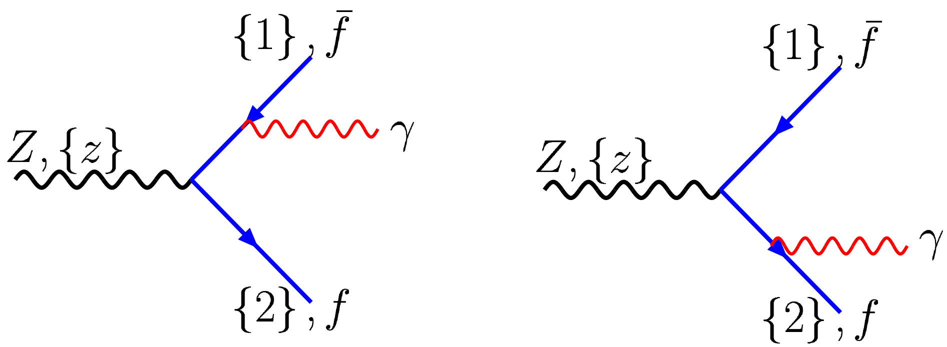


Fig. 1: Feynman diagrams for the decay $Z \to f\bar{f}\gamma$

where $\{z\} \equiv (p_z, m_z, \lambda_z)$, $\{1\} \equiv (p_1, m_1, \lambda_1)$ and $\{2\} \equiv (p_2, m_2, \lambda_2)$
For the decay amplitudes we get:

$$\begin{aligned} M^{\sigma}_{\lambda_z,\lambda_1,\lambda_2}(k,p_z,p_1,p_2) &= \left(\frac{eQ_f}{2\,k.\,p_2} b_\sigma(k,p_2) - \frac{eQ_f}{2\,k.\,p_1} b_\sigma(k,p_1)\right) B^{\lambda_z}_{\lambda_2,\lambda_1}(p_2,p_z,p_1) \\ &+ \frac{eQ_f}{2\,k.\,p_2} \sum_{\rho=\pm} U^{\sigma}_{\lambda_2,\rho}(p_2,m_2,k,0,k,0) B^{\lambda_z}_{\rho,-\lambda_1}(k,p_z,p_1) \\ &- \frac{eQ_f}{2\,k.\,p_1} \sum_{\rho=\pm} B^{\lambda_z}_{\lambda_2,-\rho}(p_2,p_z,k) U^{\sigma}_{-\rho,-\lambda_1}(k,0,k,0,p_1,m_1) \end{aligned} \tag{16}$$

where $B^{\lambda_z}_{\lambda_2,\lambda_1}(p_2,p_z,p_1) = \dfrac{g}{2\cos\theta_w} U^{\lambda_z}_{\lambda_p,\lambda_1}(p_2,m_2,p_z,m_z,p_1,m_1)$ denotes the Born-like amplitude.

$W \to f_2\bar{f}_1\gamma$

For $W \to f_2\bar{f}_1\gamma$ decay there is an one additional diagram shown in Fig.3.

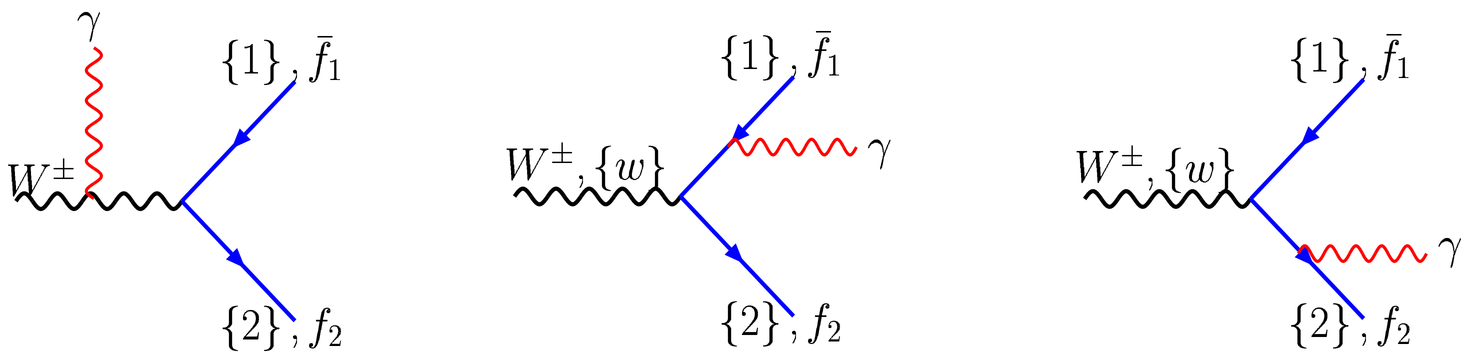


Fig.3: Feynman diagrams for the decay $W \to f_2\bar{f}_1\gamma$

where $\{w\} \equiv (p_w, m_w, \lambda_w)$, $\{1\} \equiv (p_1, m_1, \lambda_1)$ and $\{2\} \equiv (p_2, m_2, \lambda_2)$
The corresponding decay amplitudes read

$$\begin{aligned} M^{\sigma}_{\lambda_w,\lambda_1,\lambda_2}(k,p_w,p_1,p_2) \;=\; & \left[\left(\frac{eQ_{f_2}}{2\,k.\,p_2} - \frac{eQ_w}{2\,k.\,p_w}\right) b_\sigma(k,p_2) - \left(\frac{eQ_{f_1}}{2\,k.\,p_1} + \frac{eQ_w}{2\,k.\,p_w}\right) b_\sigma(k,p_1)\right] B^{\lambda_w}_{\lambda_2,\lambda_1}(p_2,p_w,p_1) \\ & + \left(\frac{eQ_{f_2}}{2\,k.\,p_2} - \frac{eQ_w}{2\,k.\,p_w}\right) \sum_{\rho=\pm} U^{\sigma}_{\lambda_2,\rho}(p_2,m_2,k,0,k,0) B^{\lambda_w}_{\rho,-\lambda_1}(k,p_w,p_1) \\ & - \left(\frac{eQ_{f_1}}{2\,k.\,p_1} + \frac{eQ_w}{2\,k.\,p_w}\right) \sum_{\rho=\pm} B^{\lambda_w}_{\lambda_2,-\rho}(p_2,p_w,k) U^{\sigma}_{-\rho,-\lambda_1}(k,0,k,0,p_1,m_1) \end{aligned} \tag{16}$$

where $B^{\lambda_w}_{\lambda_2,\lambda_1}(p_2,p_w,p_1) = \frac{g}{2\sqrt{2}} U^{\lambda_w}_{\lambda_p,\lambda_1}(p_2,m_2,p_w,m_w,p_1,m_1)$ denotes the Born-like amplitude.

The Phase Space Generation Algorithm

The Lorentz invariant phase space element for two fermions plus a photon has the form:

$$d\Phi_3(P,p_1,p_2,k) = \frac{1}{32\pi^5}\,\frac{d^3p_1}{2\,p_1^0}\,\frac{d^3p_2}{2\,p_2^0}\,\frac{d^3k}{2\,k^0}\,\delta^4(P-p_1-p_2-k) \tag{17}$$

where p_2,p_1 are fermion and anti-fermion momenta respectively, k is a photon momentum and $P = p_1 + p_2 + k$ denotes the momentum of the decaying particle.

It is clear that 3–particle phase space may be parameterized with the five independent variables. In order to take into account the peaking structure of the integrand (squared matrix element) one has to choose these variables properly. Due to the infrared and collinear divergences we have chosen the photon energy and its angle with respect to fermion as integration variables, thus improving the efficiency of Monte Carlo generation:

$$d\Phi_3(P,p_1,p_2,k) = \frac{1}{2^{10}\pi^5}\,\frac{\lambda^{1/2}(M^2,0,s)}{M}\,\frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{s}\,dk^0\,d\cos\theta_\gamma\,d\phi_\gamma d\cos\theta_2\,d\phi_2 \tag{18}$$

- k^0 – the photon energy in the rest frame of the decaying particle.
- θ_γ,ϕ_γ – the angles defining the $\vec{k}$ direction in the rest frame of the decaying particle.
- θ_2,ϕ_2 – the angles defining the $\vec{p_2}$ direction with respect to $\vec{k}$ in the rest frame of the (p_1,p_2) system.

Here M is mass of the decaying particle, $s = (p_1+p_2)^2 = M^2 - 2Mk^0$ – invariant mass of the (p_1,p_2) system and $\lambda(x,y,x) = x^2+y^2+z^2-2xy-2zx-2zy$.

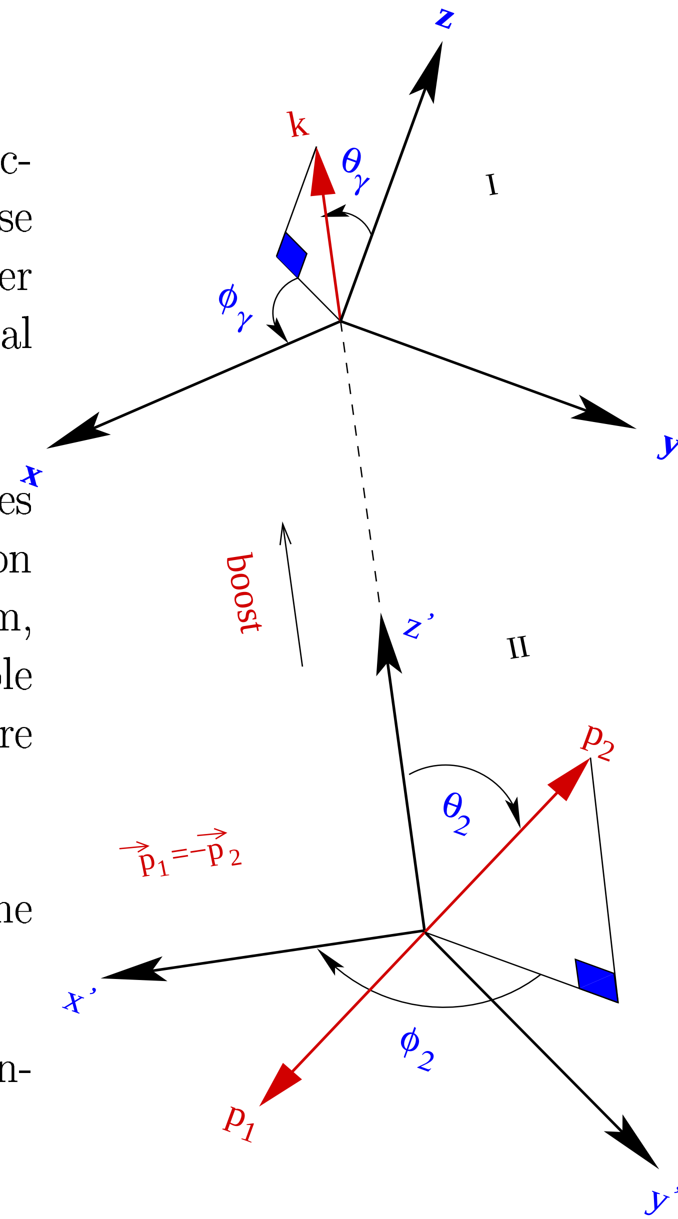
The Monte Carlo event generation procedure goes as follow:

1... In the first step we generate the photon with energy k^0 and direction $\cos\theta_\gamma$,ϕ_γ in the rest frame of the decaying particle. Because of infrared singularity k^0 is generated with a distribution $1/k^0$ over an interval (ω, k^0_{max}), where ω is a soft cutoff and k^0_{max} – maximal photon energy.

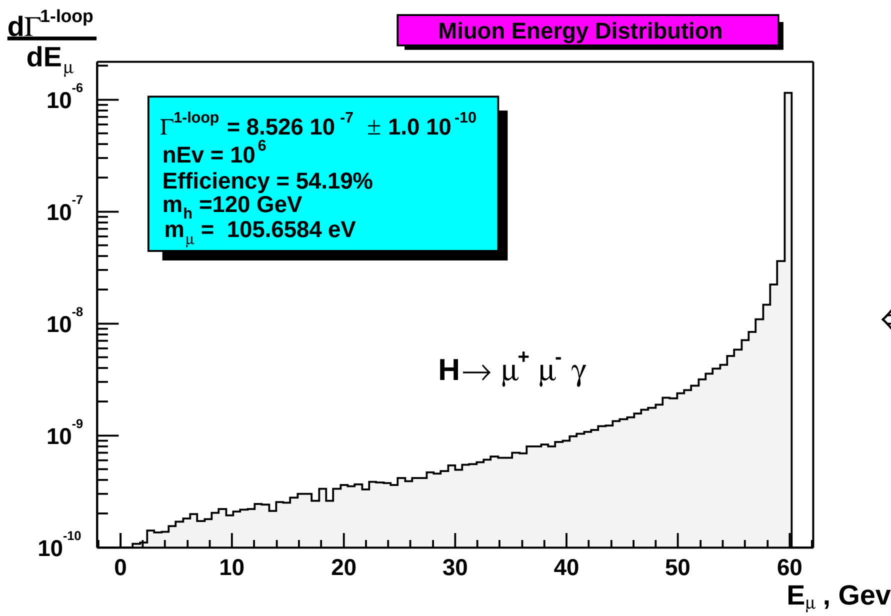
2... In the second step the fermion — anti-fermion pair with energies $p_1^0 = (s+m_1^2-m_2^2)/\sqrt{4s}$, $p_2^0 = (s+m_2^2-m_1^2)/\sqrt{4s}$ and direction $\cos\theta_2$,ϕ_2 are generated in the rest frame of the (p_1,p_2) system, where due to collinearity (light fermions) we generate the variable $cos\theta_2$ with a distribution $1/(1-\beta_1\cos\theta_2)+1/(1+\beta_2\cos\theta_2)$, here $\beta_{1,2} = |\vec{p}_{1,2}|/p^0_{1,2}$.

3... Further the p_1 and p_2 momenta are boosted from their rest frame to the rest frame of the decaying particle along the $\vec{k}$ direction,

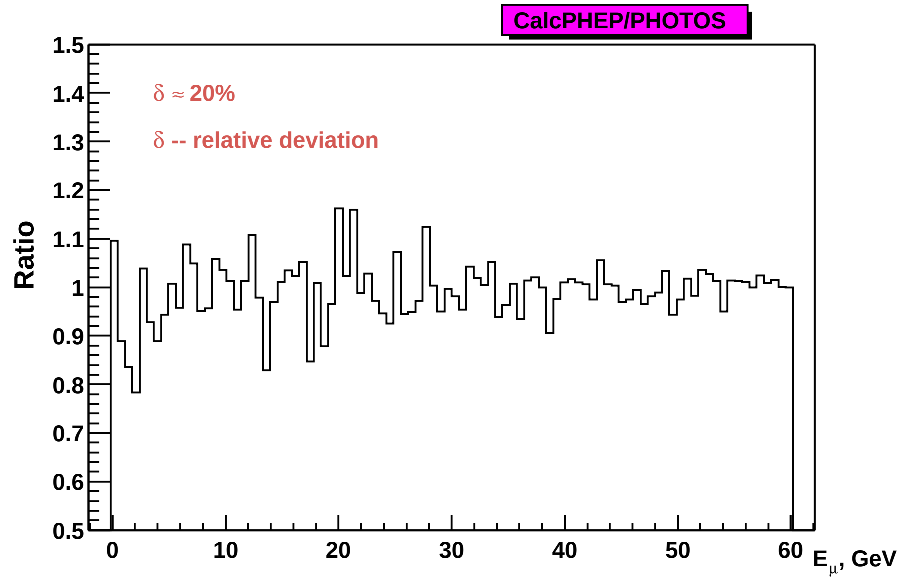
4... and finally, we rotate $\vec{p}_1$ and $\vec{p}_2$ vectors respecting momentum conservation $\vec{k}+\vec{p}_1+\vec{p}_2 = 0$.

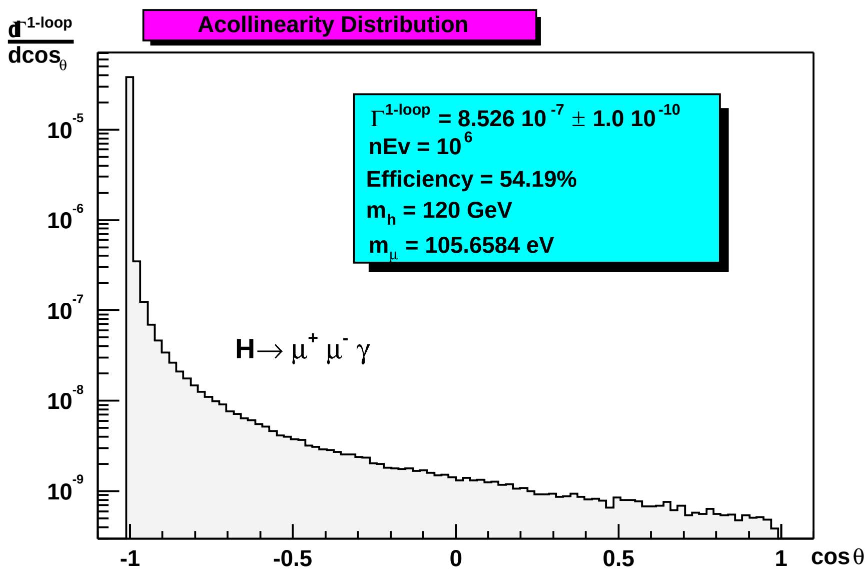


This figure illustrates the event ge neration procedure described above.
I – the rest frame of the decaying particle.
II – the rest frame of the (p_1,p_2) system.

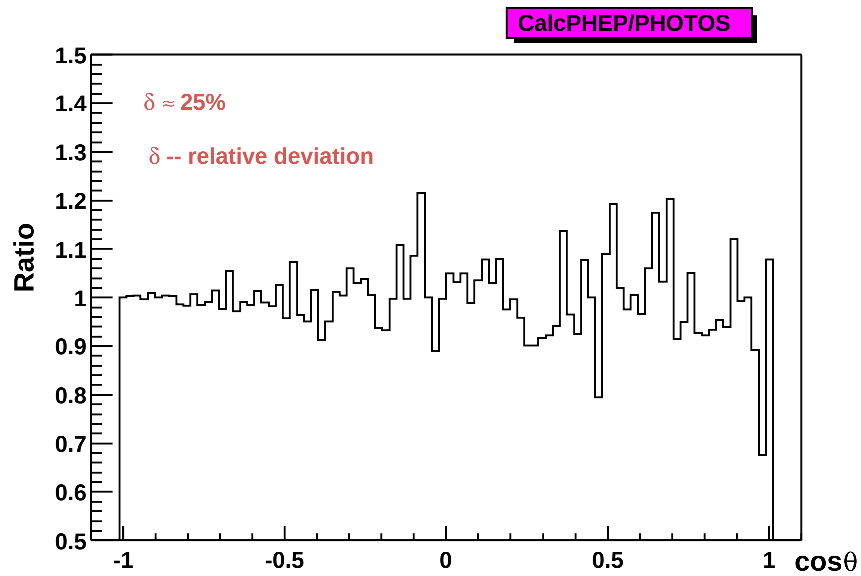


$\Leftrightarrow$



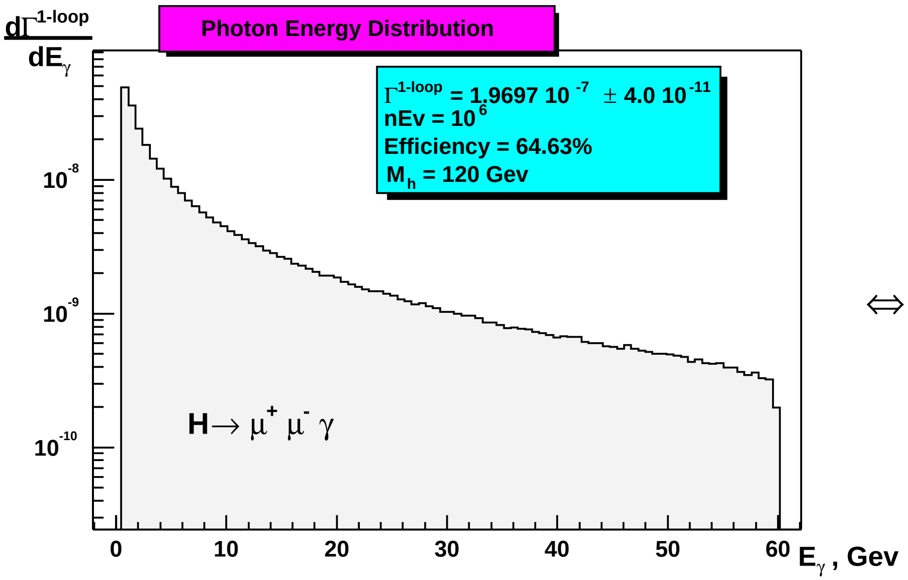


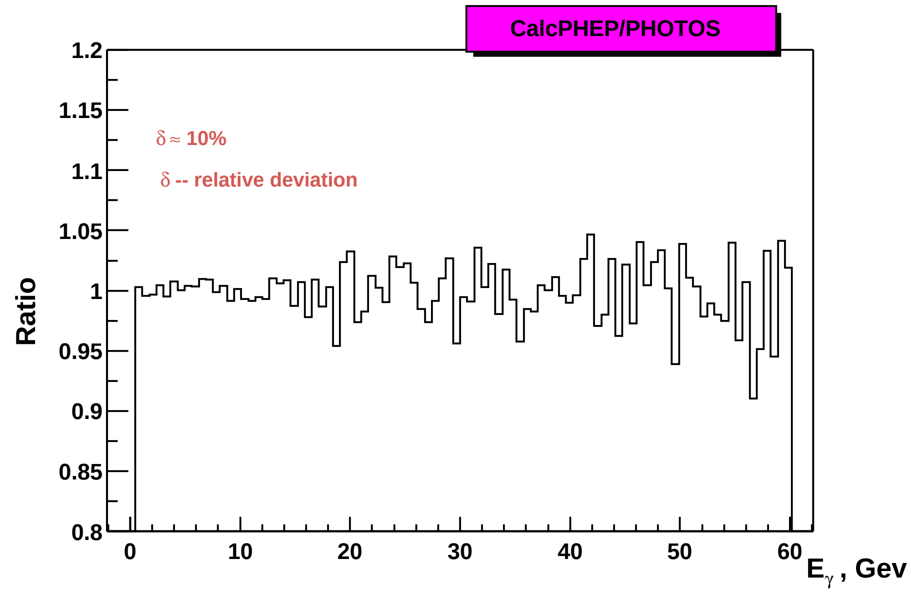
$\Leftrightarrow$

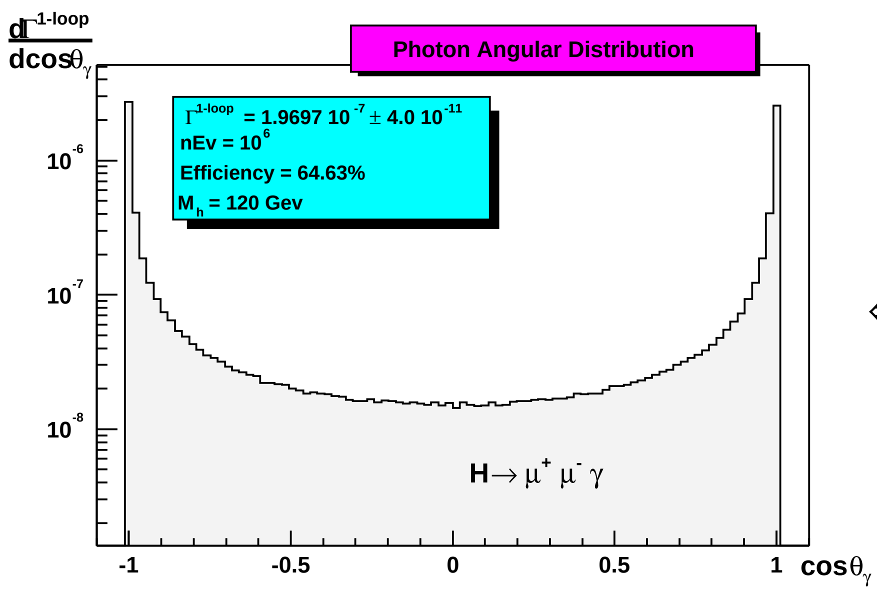


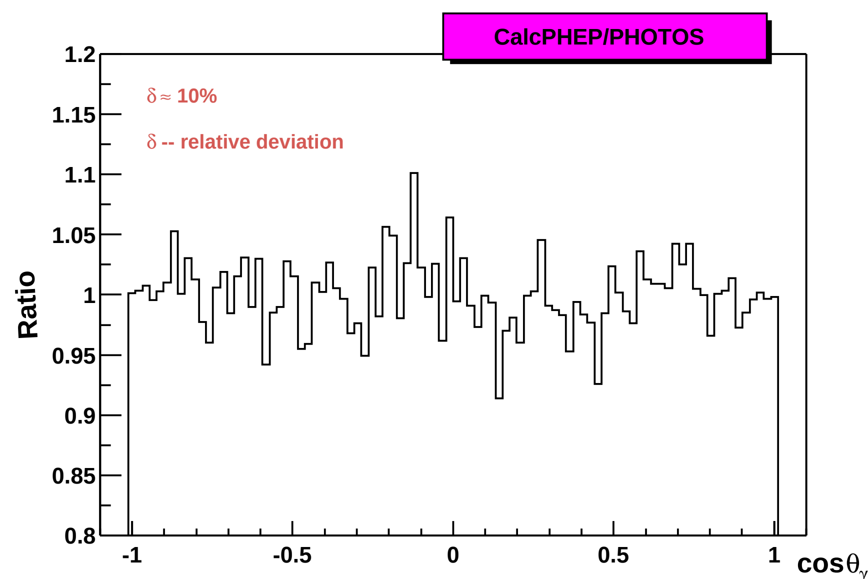
The CalcPHEP Monte Carlo rezults for miuon energy and acollinearity distribution in decay $H \to \mu^+\mu^-\gamma$.

Comparison with PHOTOS



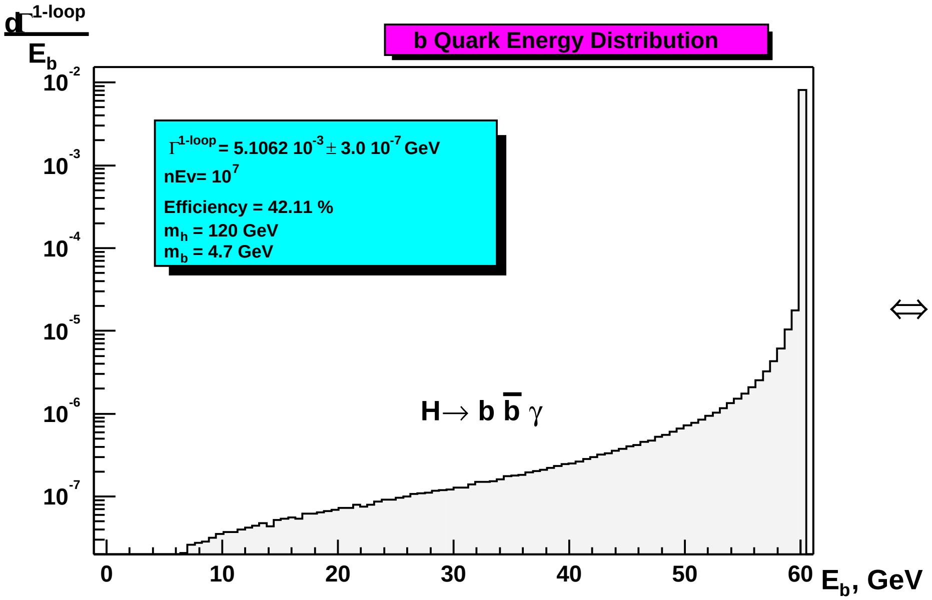




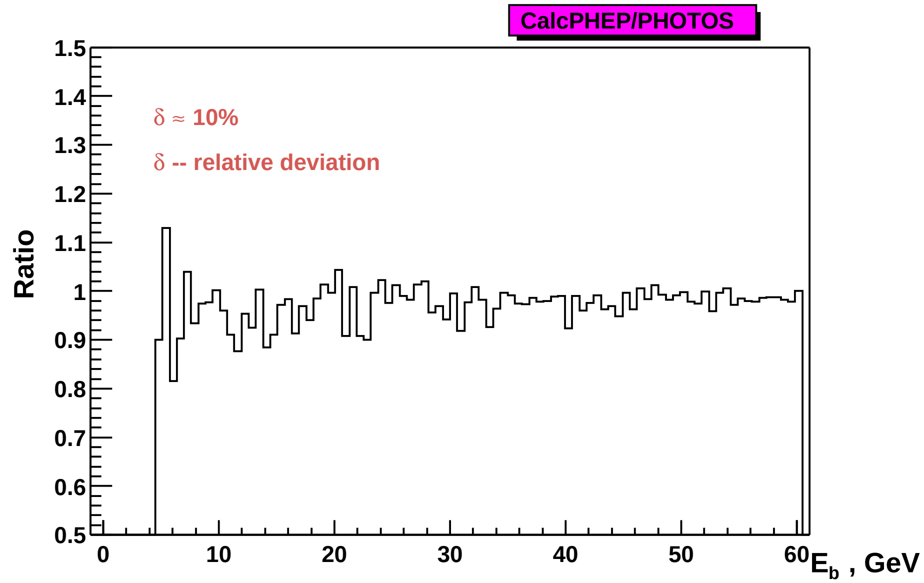


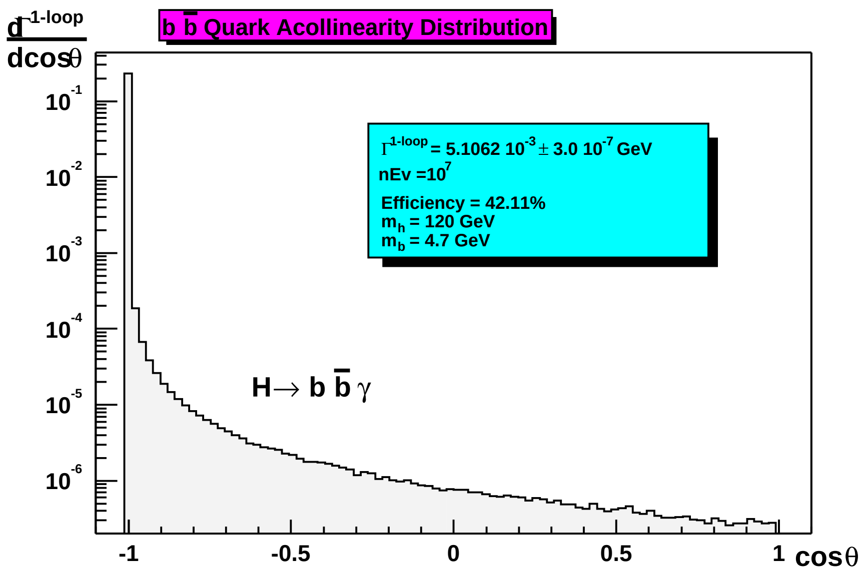
The CalcPHEP Monte Carlo rezults for photon energy and angular distribution in decay $H \to \mu^+\mu^-\gamma$.

Comparison with PHOTOS

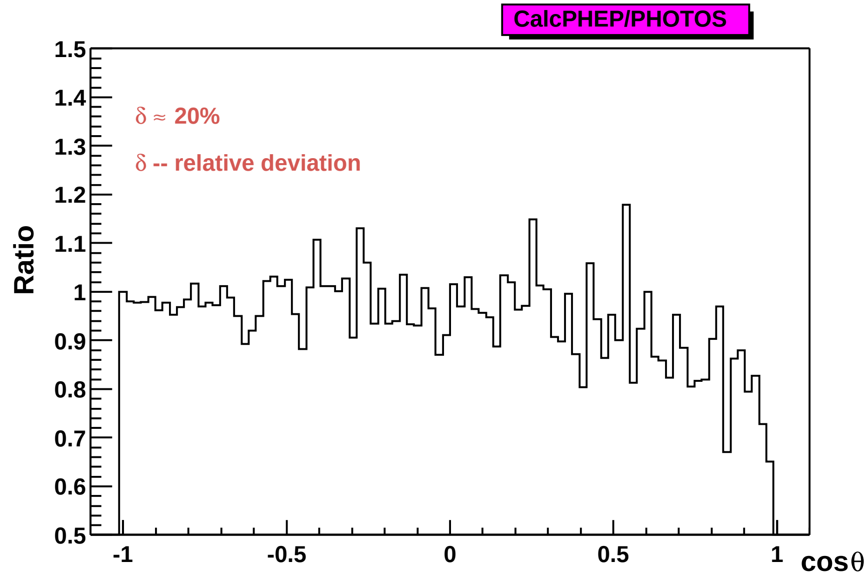


⇔



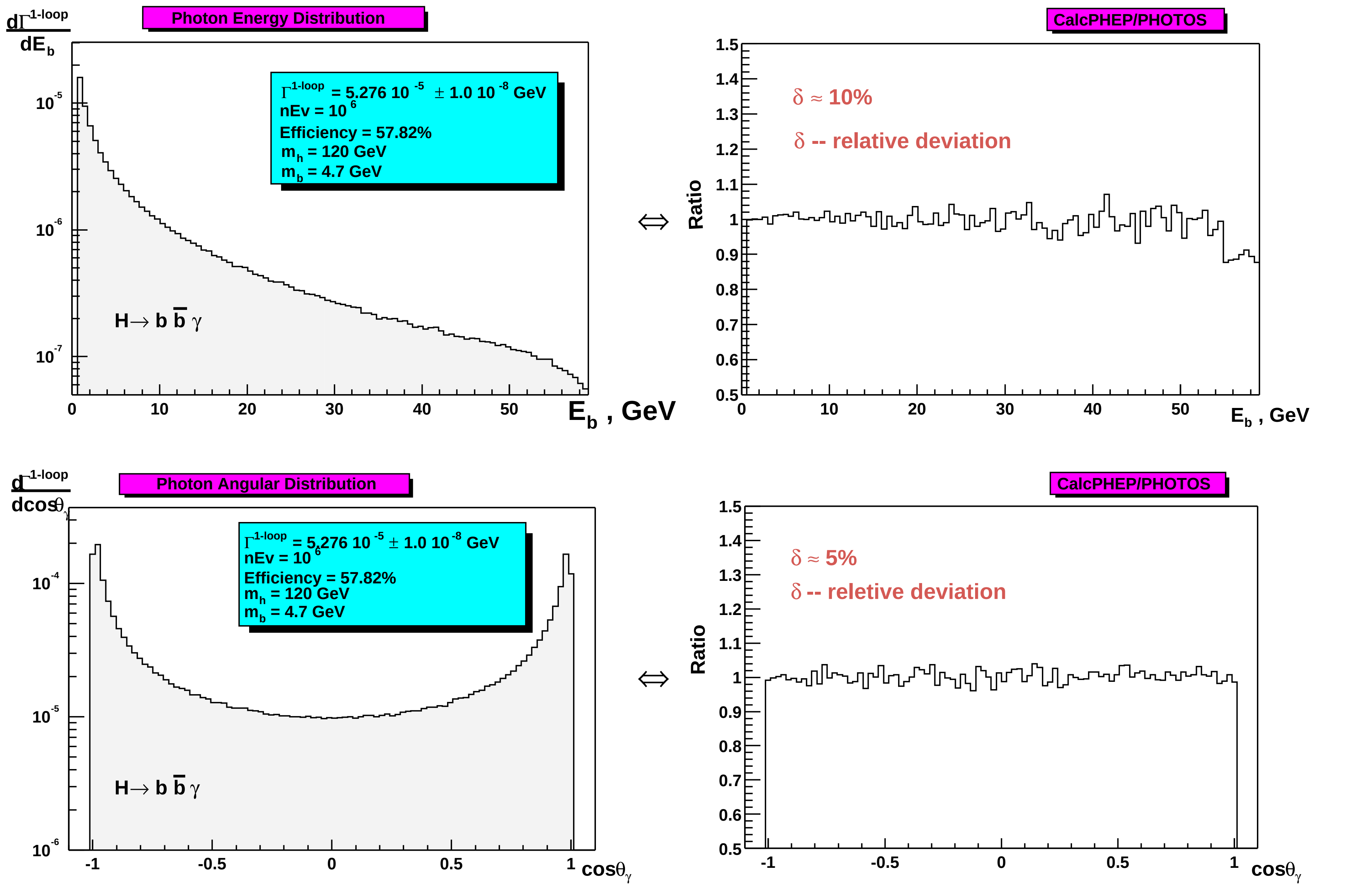


⇔



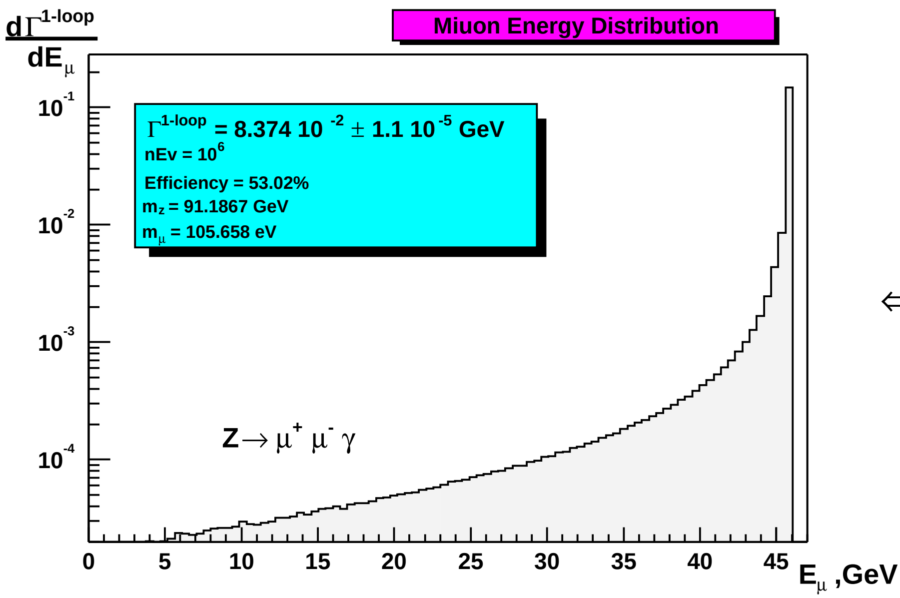
The CalcPHEP Monte Carlo rezults for b quark energy and acollinearity distribution in decay $H \to b\bar{b}\gamma$.

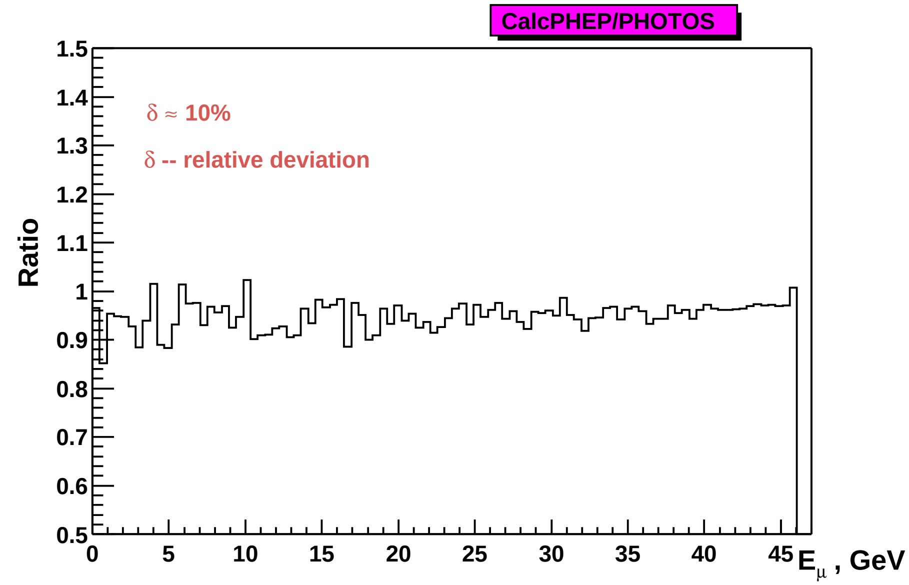
Comparison with PHOTOS

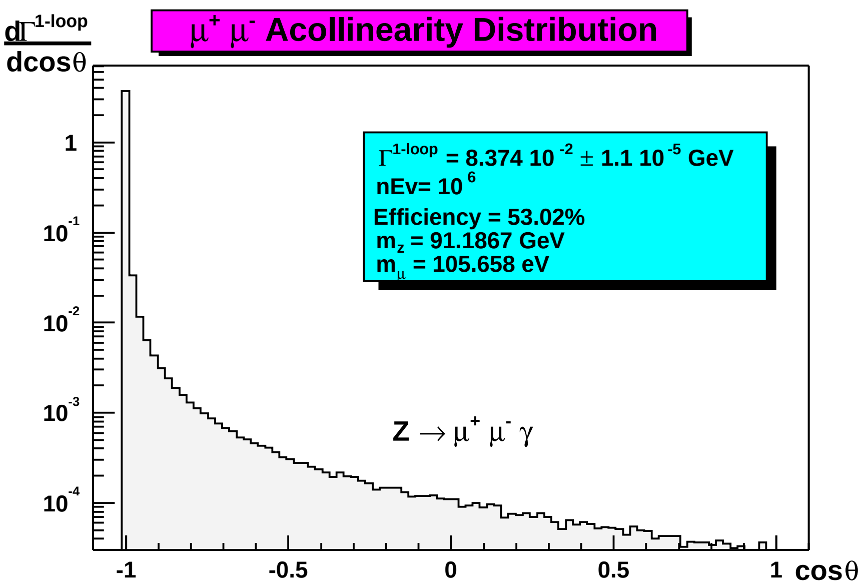


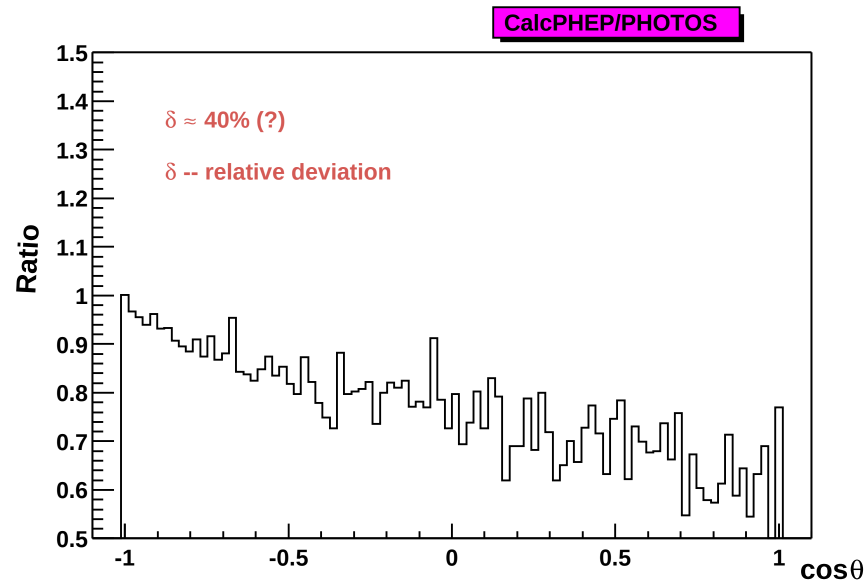
The CalcPHEP Monte Carlo rezults for photon energy and angular distribution in decay $H \to b\bar{b}\gamma$.

Comparison with PHOTOS



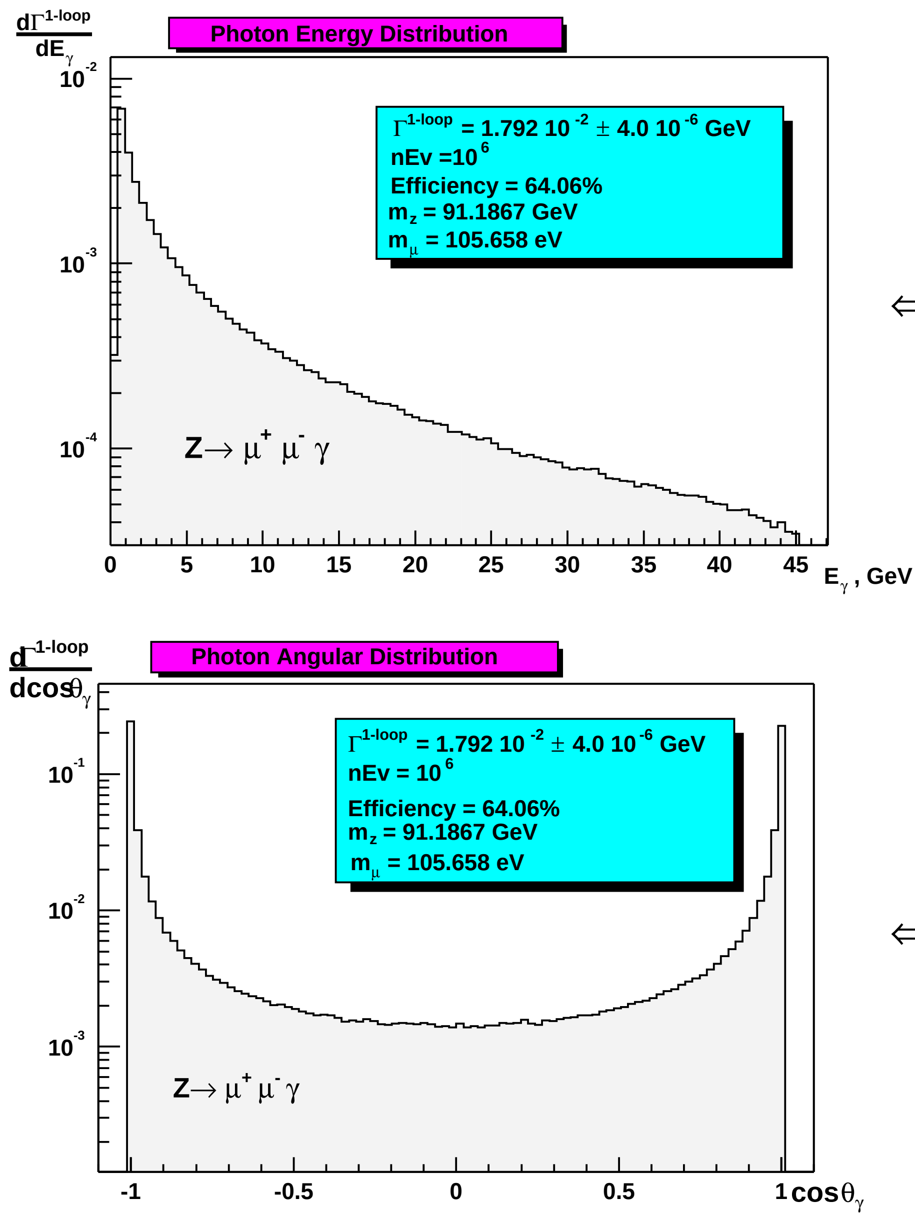






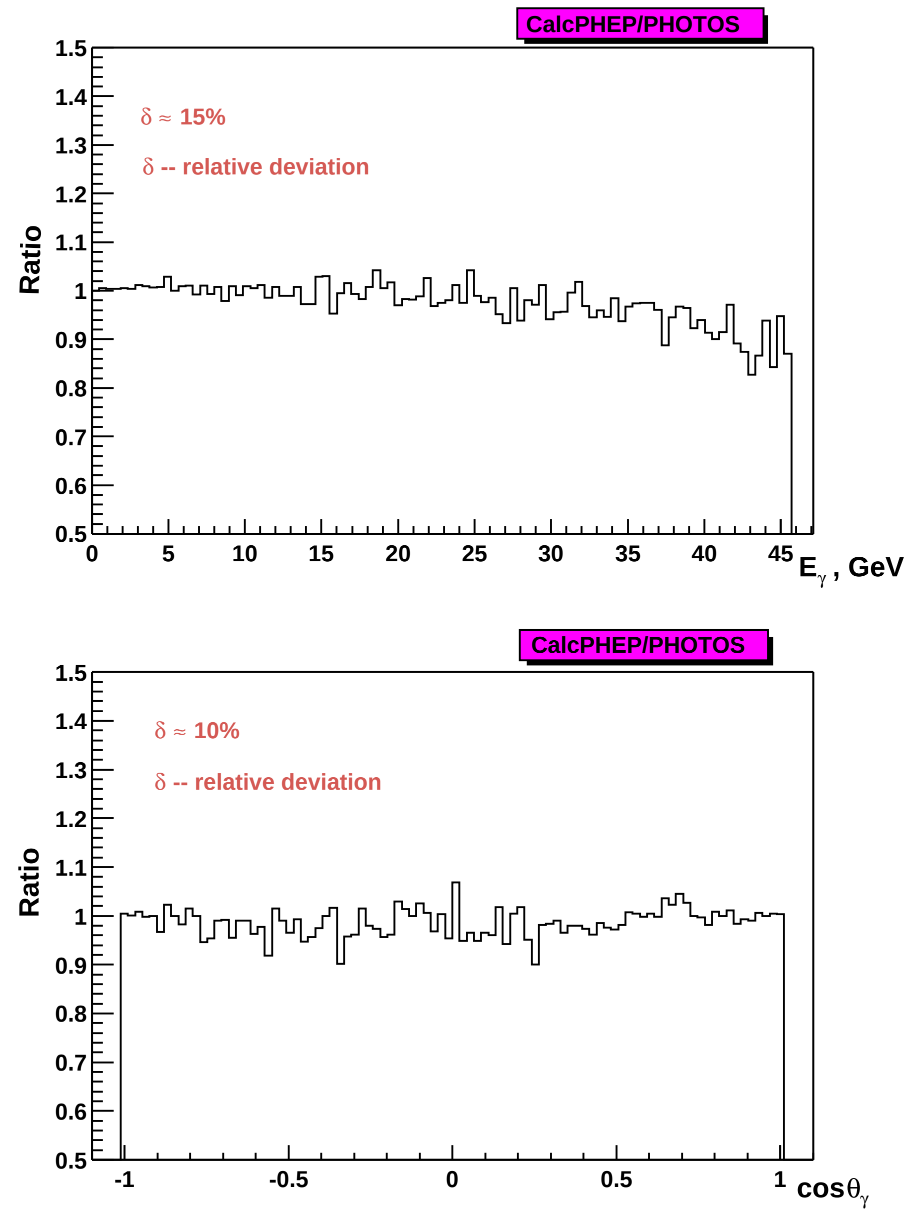
The CalcPHEP Monte Carlo rezults for miuon energy and acollinearity distribution in decay $Z \to \mu^+\mu^-\gamma$.

Comparison with PHOTOS

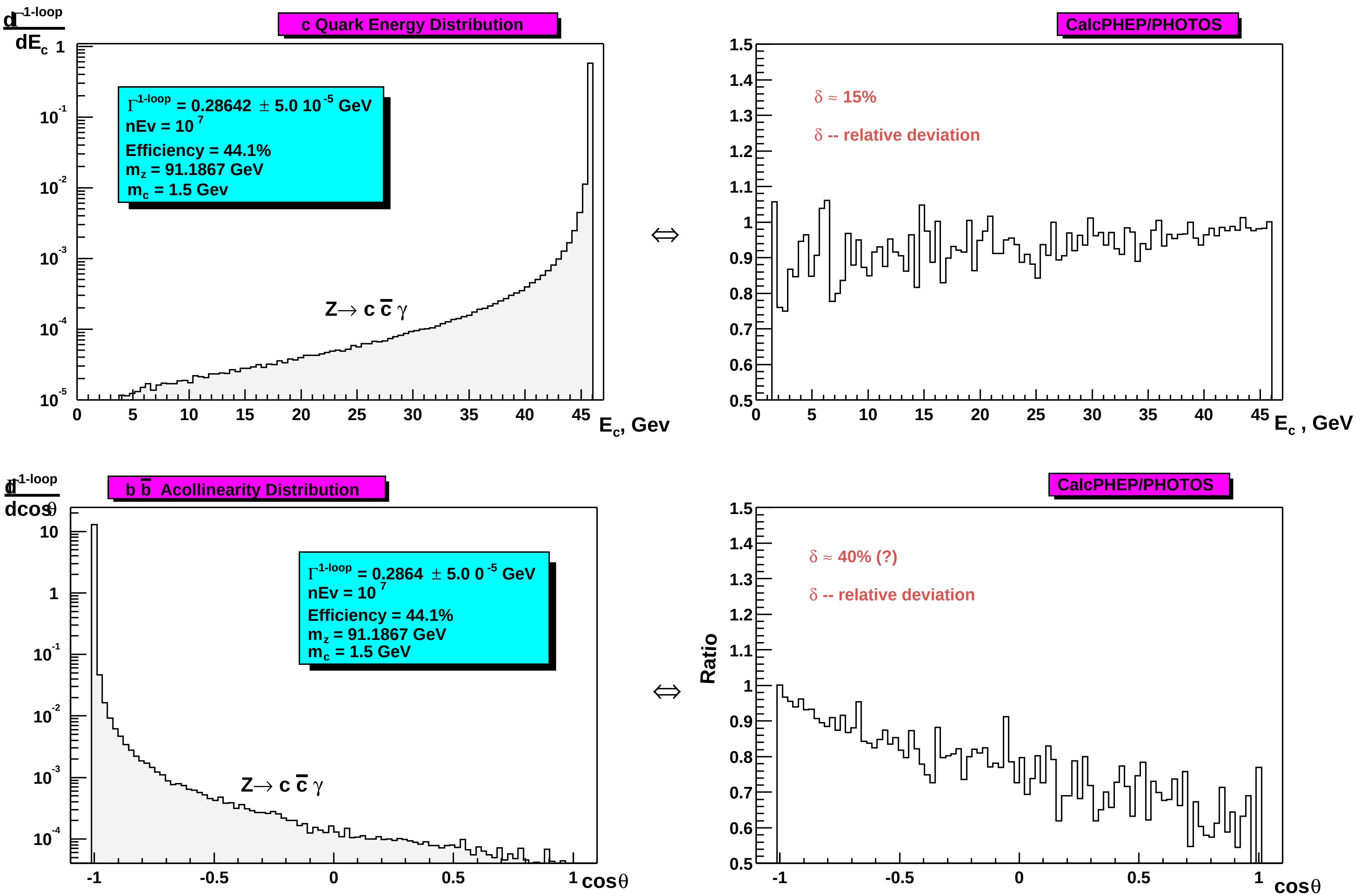


The CalcPHEP Monte Carlo rezults for photon energy and angular distribution in decay $Z \to \mu^+\mu^-\gamma$.

$\Leftrightarrow$

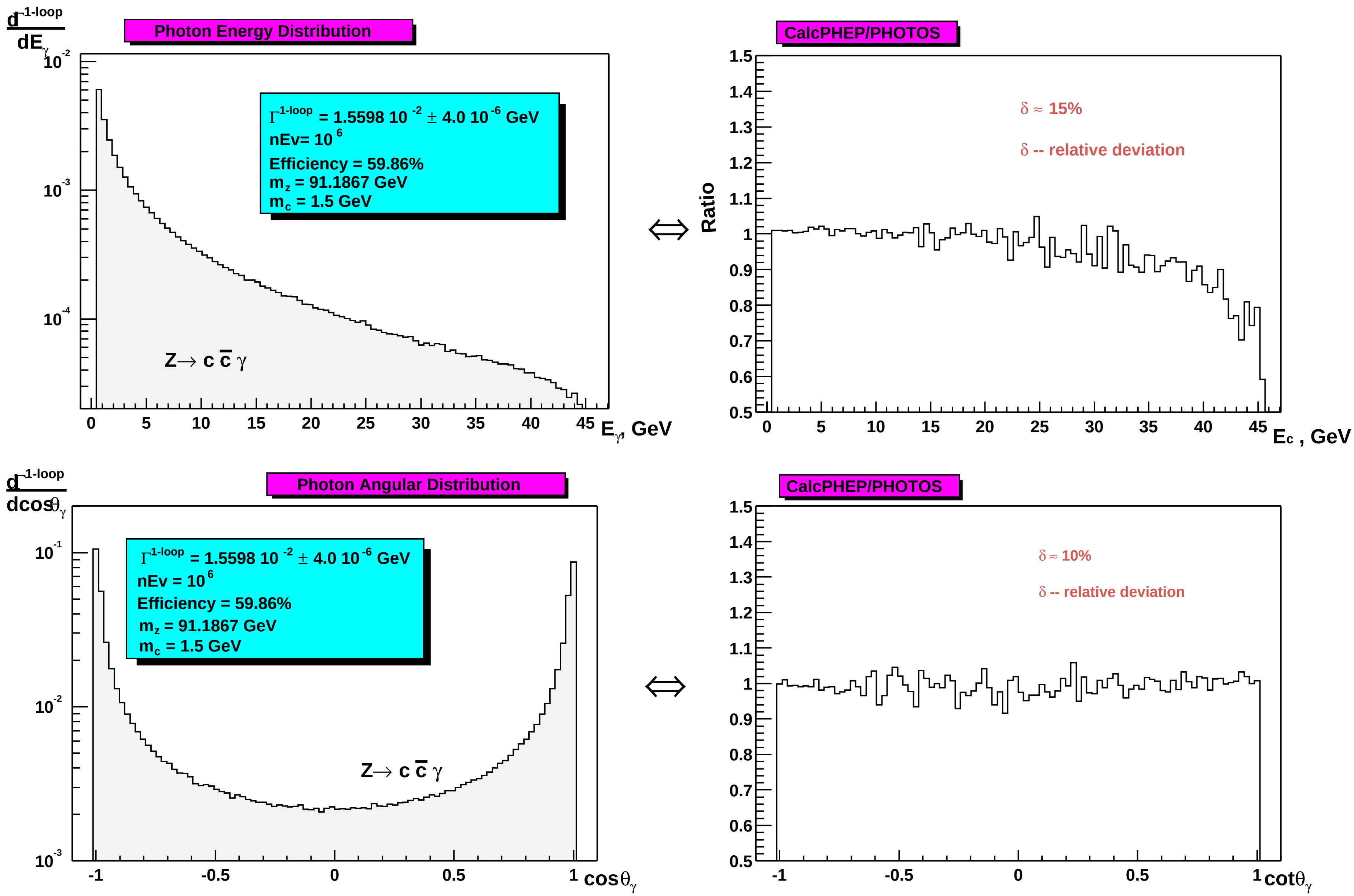


Comparison with PHOTOS



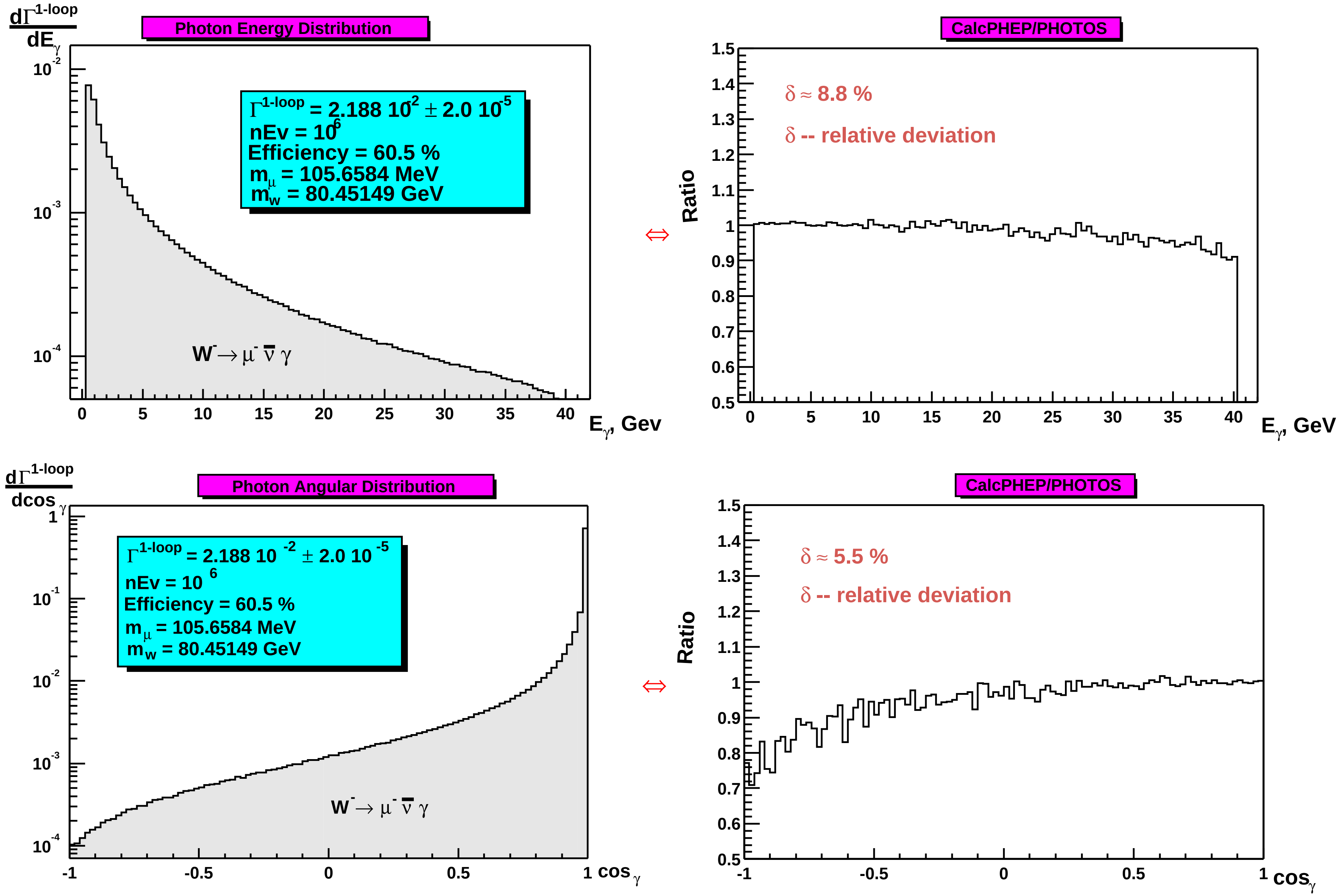
The CalcPHEP Monte Carlo rezults for miuon energy and acollinearity distribution in decay $Z \to c\bar{c}\gamma$.

Comparison with PHOTOS



The CalcPHEP Monte Carlo rezults for photon energy and angular distribution in decay $Z \to c\bar{c}\gamma$.

Comparison with PHOTOS



The CalcPHEP Monte Carlo rezults for photon energy and angular distribution in decay $W^- \to \mu^- \bar{\nu}\gamma$.

Comparison with PHOTOS